

COMPARATIVE ANALYSIS OF VISCOUS AND INVISCID FLOW IN AERODYNAMICS

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Abstract— Aerodynamic analysis often relies on simplified flow models to predict lift and drag on aerodynamic surface. Inviscid flow theory, governed by the Euler equations, provides useful physical insight but neglect the viscous effects which cannot be avoided in the real-life incidents. This paper presents a comparative analysis of the way which the viscous and inviscid flow models behave around an airfoil. The fundamental assumptions are reviewed while discussion of the Euler and Navier–Stokes equations. Moreover, the study demonstrates that inviscid models are valuable for preliminary and theoretical understanding, while viscous flow modeling is required for accurate aerodynamic predictions though it reduces the efficiency of the experimental process. To combine both efficiency and accuracy, a hybrid modeling approach is used.

Keyword— Aerodynamics; Airfoil; Inviscid Flow; Viscous Flow; Euler Equations; Navier–Stokes Equations; Boundary Layer; Wall Shear Stress; Flow Separation; Lift; Drag; Vorticity; Energy Dissipation; Hybrid Modeling;

I. INTRODUCTION

The field of aviation has experienced rapid growth over the past few decades by achieving significant milestones. However, in the early stages of aviation, pioneers relied heavily on trial-and-error methods to understand how aircraft could generate lift and maintain stability. As a result of aircraft designs became complex over time, the need of scientific understanding of airflow and aerodynamic forces became extremely essential. Aerodynamics is a fundamental branch of fluid mechanics which studies about the way which airflow behaves when interacting with solid surfaces, particularly aircraft wings and other aerodynamic bodies. The accurate modeling of airflow is crucial to predict the aerodynamic forces such as drag and lift which mainly rely on aircraft's performance, stability and efficiency. Usually in aerodynamic analysis, airflow is simplified using theoretical models in order to make calculations more manageable. Two such widely used models are inviscid flow (which ignores the viscosity of the fluid) and viscous flow (which provides additional account for internal friction in the fluid). Although the inviscid flow theory gives insight into idealized flows, it fails to explain some aerodynamic phenomena often seen in

the real world, including drag generation and boundary layer formation. Here in this paper the main aim is to compare viscous and inviscid flow models and analyze their effects on aerodynamic surfaces while examining their assumptions predictions and limitations. And highlights the importance of viscosity when performing realistic and accurate aerodynamic analysis.

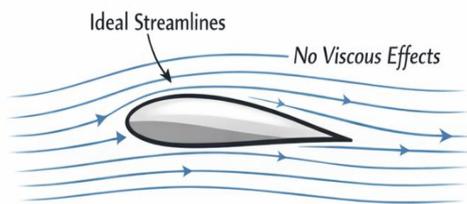
Note to Readers: In this paper, the terms "inviscid flow" and "Euler equations" describe idealized fluid behavior to help with understanding concepts. Inviscid flow ignores viscosity and wall shear. This simplifies calculations but does not reflect all real-world aerodynamic effects. Assumptions like incompressibility or irrotationality only hold true in specific situations, such as subsonic, streamlined flows. Viscous effects, including boundary layer formation, skin-friction drag, vorticity, and energy loss, are important for making accurate predictions about real airflow. Readers should see inviscid results as idealized approximations, not as exact behavior in the real world.

INVISCID FLOW

The evolution of aerodynamic theories extends from observations of airflow around simple objects and foundational mathematical principles to complex computational models. However, the concepts of aerodynamic were pioneered by Bernoulli in 18th century, which gave us the initial concept about the relationship between fluid velocity and pressure. Meanwhile, with the formulation of Navier Stokes equations which describes the motion of viscous fluid, the 19th and 20th centuries saw significant developments and laid the foundation of modern fluid mechanics. Meanwhile the practical experimental demonstrations by aviation pioneers such as Wright brothers contributed for the development of theories. The integration of theoretical and practical helped scientists identifies the differences between inviscid and viscous flows.

Inviscid flow is the idealized model of fluid motion in which the based on several assumptions. Primarily the effects of viscosity are neglected. In simple terms, there is no internal friction, which allows the fluid to behave as a perfectly slippery medium. This analysis usually makes the analysis of airflow much easier and can be used for understanding the behavior of fluids around aerodynamic surfaces under ideal conditions. This model is commonly applied in the theoretical calculation of lift and in the design of streamlined shapes, where viscosity has a negligible effect on overall flow.

Furthermore, in this model wall shear stress is ignored, and aerodynamic drag occurred by interaction between a body surface and a fluid is also assumed to be zero. Inviscid simulations, often assume the flow is steady. This means the fluid properties at any point in the system do not change over time. Additionally, thermodynamic properties are frequently assumed to be constant throughout the flow field. For many inviscid analyses, the flow is assumed to be irrotational and Incompressible Flow (But Inviscid flow can be either compressible or incompressible. For subsonic flows, incompressible assumption is valid; for high-speed flows, compressibility must be considered). So that density is constant in many occasions. Furthermore, it is assumed that the body's motion causes only a negligible disturbance to the uniform fluid. Consequently, the validity of these mathematical theories depends on assumptions but when boundary layer separation occurs the accuracy drops.



Inviscid Flow Around an Airfoil

The Euler equation

The Euler equations describe the motion of a perfect fluid, which is inviscid. The Euler equations allow us to predict the behavior of fluid flow around the aerodynamic surface, the fluid is inviscid. However, Viscosity has to be ignored totally. The Euler's fundamentals of the equations rely on Newton's second law. So that the change of momentum of fluid particles equal to the forces acting on it.

$$m \cdot \frac{D\vec{v}}{Dt} = \text{sum of forces (pressure forces + body forces)}$$

$$m = \rho dV$$

$$\text{body forces (like gravity)} = \int \rho dV$$

$$\text{pressure forces} = -\nabla p dV$$

(usually, the fluid appears from high pressure to low, - sign is added)

$$\rho dV \cdot \frac{D\vec{v}}{Dt} = -\nabla p dV + \int \rho dV$$

divided by volume(dv)

$$\rho \frac{D\vec{v}}{Dt} = -\nabla p + \int \rho$$

VISCOUS FLOW

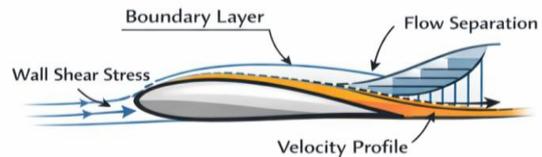
In fluid dynamics, viscous flow refers to a regime where the fluid's viscosity can not be neglected. Unlike inviscid models, viscous analysis accounts for the complex interactions between the solid surface and fluid particles. Presence of internal friction, which is characterized by viscosity; leads to several physical phenomena which cannot be seen in inviscid flow.

• Wall shear stress:

In the presence of viscosity, the interaction between the fluid and the solid surface causes wall shear stress. Surface roughness contributes this effect stronger by increasing friction between the fluid layers and the solid boundary. Both the normal stress arises due to pressure and wall shear stress start to act on aerodynamic surface simultaneously, causing a critical role in determining the total aerodynamic forces acting on a body, especially drag. Unlike inviscid flow models, viscous flow analysis directly considers these surface-related forces leading to get a proper scientific understanding with more accuracy.

• No-Slip Condition and Boundary Layer Formation:

No-slip condition is one of the key assumptions of viscous flow, which means particles in contact with the solid surface have zero velocity relative to that surface. The relative velocity the fluid particles next to the nonmoving particles relative to the surface, gradually increases from zero at wall to the free-stream value free-stream value relative to the surface, resulting a boundary layer. This boundary layer is a thin region with a large velocity gradient as well as it causes aerodynamic effects including skin-friction drag and flow separation.



Viscous Flow and Boundary Layer

• Vorticity Generation and Energy Dissipation:

Usually when air flows over the solid aerodynamic face, air particles closer to the surface moves more slowly due to friction. This difference in speed between air layers in different levels leads air to rotate and swirl. This is called as vorticity. Mainly this rotation occurs inside the boundary layer near the surface due to presence of a large velocity gradient. Due to presence of friction between the air layers and the surface, some of air flow's mechanical energy is lost as heat. This loss of energy is known as energy dissipation. The energy lost by the frictional force contributes aerodynamic drag which negatively contributes to the aircraft's motion directly

effects on aircrafts efficiency. Furthermore in such cases some basic principles such as Bernoulli's principle which relies on Law of energy conservation cannot be applicable. So that considering velocity generation and energy dissipation is important to get an idea about why real airflow loses energy and creates drag. It allows modern day aviation engineers to predict real aerodynamic behaviors with high rate of accuracy.



Vorticity and Energy Dissipation

Despite from the advantage of analysis of viscous flow, it has several boundaries since the consideration of the concept viscosity paves the path for complex governing principle and theories. Primarily in most practical instances it increases the complexity, forcing to use numeric methods and computational simulations instead of solving analytically. Though it shows more accuracy than inviscid flow models, viscous flow is more complicated and challenging.

The Navier-Stokes equations

The Navier-Stokes equations describe momentum conservation in viscous fluids. According to Newton's second law $mass \times acceleration = sum\ force$

Where,

$$mass = \rho dV$$

$$acceleration = \frac{DV}{Dt}$$

$$Sum\ of\ forces = Pressure\ forces + Viscous\ forces + Body\ Forces$$

Where,

$$pressure\ forces\ per\ unit\ volume = -\nabla p$$

$$viscous\ forces\ per\ unit\ volume = \mu \nabla^2 \vec{v}$$

$$body\ forces\ per\ unit\ volume = \rho \vec{f}$$

$$So\ that, \rho \frac{DV}{Dt} = -\nabla p + \mu \nabla^2 \vec{v} + \rho \vec{f}$$

II. COMPARISION OF VISCOUS AND INVISCID FLOW

COMPARISION BE

1. Physical Assumption

The core difference between viscous and inviscid flows is lies on the way of consideration for internal friction. When the flow is inviscid there is no viscosity at all, consequently the presence of the internal friction and wall shear stress are completely ignored. Moreover, the fluid particles are permitted to slide over the solid surface without any

interference. Meanwhile viscosity imposes the nonslip condition so that velocity on fluid body is zero relative to the surface, allowing to formation boundary layer.

2. Aerodynamic Forces and Drag

Sometimes two models produce vastly different predictions regarding the behavior of aerodynamic forces

- Drag prediction: In a 100% inviscid model, theoretically drag coefficient is zero. And there is no wall shear. Simultaneously for viscous modeling it is necessary to consider the skin friction (by surface roughness and shear stress) and form drag (by pressure distribution around the body).

- Lift: Inviscid models give reasonable predictions and estimations about aircraft's lift at low angles of attack. However, when angle of attack increases, automatically reduces its accuracy due to viscous-induced flow separation.

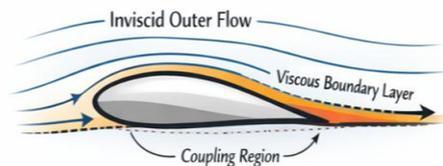
- Inviscid models ignore viscosity. But in reality, actual behavior is different since viscosity cause the formation of boundary layer, this effect leads to change the apparent stiffness of the tail so that the tail feels more resistances to flutter and causes an abnormal behavior than the predictions given by the inviscid flow

3. Flow Separation and Complexity

The inviscid theory is incapable to handle flow separation properly. Hence inviscid models generally consider that flow remains fully attached to the body. However, the point where the fluid where fluid detached from the surface is completely viscous phenomenon governed by the boundary characteristics.

Hybrid Viscous-Inviscid Interaction

As some expensive computations have to be done in order to solve the full viscous equations for an entire flow field, Researchers often use Viscous-Inviscid Interaction, here in the external region where velocity potential and lap place equations are directly applicable the flow is considered as Inviscid, While the flow is considered as Viscous in the boundary layer. By integrating these two regions, the model can account for viscous effects, as well as maintain the computational efficiency.



Hybrid Viscous-Inviscid Interaction



Condition	Lift Coefficient (CL)	Drag Coefficient (CD)	Notes	Source / Reference
Inviscid Flow	0.65	0.001	Drag negligible, idealized flow	Anderson, 2010
Viscous Flow	0.62	0.009	Boundary layer effects included	Wu et al., 2018
Hybrid Model	0.63	0.005	Combines efficiency and accuracy	Rodriguez, 1993

The table shows comparison of lift and drag coefficients for different flow models.

III. CONCLUSION

The study of aerodynamic flows demonstrates that the viscosity shows a vital role in predicting the behavior of fluids around aerodynamic surfaces with more accuracy. Inviscid flow models neglect internal friction and provide simplified mathematically convenience approach in order to analyze the lift and other flow patterns. They are particularly useful for conceptual understanding. However, in the real world the effect of viscosity cannot be avoided. Viscous phenomena such as boundary layer formation, wall shear stress, vorticity, energy dissipation, and aerodynamic drag cannot be studied by using inviscid models. Though Viscous flow principals make the task more complex, allows engineers to predict the air flow more accurately. Therefore, a combined model is used to maintain the accuracy as well as the efficiency.

Greek letters

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Symbol	Meaning / Description
ρ (rho)	Fluid density
μ (mu)	Dynamic viscosity
$\mathbf{V} \rightarrow$	Velocity vector
p	Pressure
$\mathbf{f} \rightarrow$	Body force per unit volume (e.g., gravity)
t	Time
DV/Dt	Material (total) derivative of velocity
∇ (nabla)	Gradient operator
∇^2 (Laplacian)	Laplace operator, appears in viscous term of Navier–Stokes equations
dv	Differential volume element
C_l	Lift coefficient
C_d	Drag coefficient
α (alpha)	Angle of attack

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