

# FORCED VIBRATION MATHEMATICAL ANALYSIS OF CHERY 100HP TRACTOR TRANSFER CASE

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**Abstract:** In this study forced vibration of Chery 100hp tractor transfer case was analyzed using mathematical modeling. The system defined as a forced vibration damped and with a single degree of freedom, all the important feature in the system has been represented, mathematical and analytical equation has been derived including governing equation by using D'Alemer's principle which is the principle of conservation energy.

The system was analyzed based on damping oil type by varying damping ratio ( $\xi$ ) between 0 to 1 between under damped to over damped to analyze resonance situation, mechanical material property and force excitation by taking 95 hp and 100hp reference. By importing the governing equation in to Mat Lab software amplitude vs. time graph was plotted by taking  $\xi=0$  with 100 hp and 95 hp,  $\xi=1$  with 100hp and 95 hp with various material type.

Its observed that the system is not disturbed on the available oil damper (SAE 140) when the excitation force is 95 hp but there will be a problem on 100hp and the same oil type. That implies one of the failure reason for the system is damping oil type.

Taking the best transfer case oil specification which are For SAE J306 is  $5.29 \times 10^{-6}$ , SAE 90 is 0.027, SAE 140 is  $5.3 \times 10^{-6}$ , SAE 250 is  $4.81 \times 10^{-6}$ , ISO 100 is  $7.44 \times 10^{-6}$ , ISO220 ,  $5.68 \times 10^{-11}$ , ISO 150 is  $5.35 \times 10^{-3}$ , ISO 68 is  $4.81 \times 10^{-6}$  and SAE 85W-140 is  $7.68 \times 10^{-6}$  and checking the damping ratio found that the value of damping factor neighboring to one( $\xi=1$ ) comparing to the others is SAE 90 oil type for this system.

Analyzing the natural frequency of material which are Aluminum Alloy 1100 is 214.38hz, Copper Alloy is 440.70hz , Gray cast Iron is 684.00 hz Steel Alloy is 805.80 hz Carbon and low steel alloy (ASTM)is 491.94 from these material steel alloy is found to the best material for the transfer casing.

## I. INTRODUCTION

The term vibration is used to include shocks and movements of any kind result from machine and terrain factors (Piersol, 2010). Vibration characteristics are determined by a structure's mass and stiffness values, with damping (ability

to dissipate vibrational energy) (Meirovitch, 2001). Forced mechanical vibrations occur when an external force continually influences a mechanical system (Rao, 2017). Tractors are sometimes designed to perform vibration on field activities intentionally Transfer cases are used in off road vehicles to divide engine torque between the front and rear driving axles. The transfer case also allows the front driving axle to be disengaged, which is necessary to prevent undue drive line component wear during highway use (M. Costa, 2016).

During power transmission if there is a slack in drive train it may cause high vibration known as transmission Shock. It is the main reason for gearbox assembly and housing failure (Rajput, 2016)

A major advantage is that vibration analysis can identify developing problems before they become too serious and cause unscheduled downtime (Randall, 2011). The analysis of vibratory system usually involves mathematical modeling derivation of the governing equation, solution of the equation and interpretation of the result. The main objective of the thesis is to analyze the forced damped vibration of Chery 100hp tractor transfer case by using mathematical analysis.

## II. MATERIAL AND METHOD

### 2.1. Material

#### 2.1.1 Transfer case

The transfer case has 36 components including shift fork shaft, intermediate gear shaft, transfer case gear shaft ,bearings, gears both driven and driving gear, sleeve, spacer bush and others figure. The gears are spur gears meshed in a constant mesh. The dimension of the transfer case is length 145mm, Width 122mm and height 185mm.



transfer case disassembled and measured using caliper and meter in order to get proper design.



**Figure 1 Measurement taken for technical specification**

### 2.1.2 Software Tools

After manual measurement has been taken with the free hand sketching of the transfer case the 3D design has been conducted with the help of CATIA V5 these has a great contribution for calculating moment of Inertia as the geometrical pattern was very complicated and calculation was made manually. Later MATLAB 2017a software helped to draw graphs with the general equation developed by mathematical analysis.

## 2.2 Method

### 2.2.1 Data Collection

Technical Specification

Technical specification was taken for the sake 3D modeling on Catia V5 soft were each component of the tractor model

### Spectrometer test

Knowing the material type is mandatory to study vibration. The sample of transfer case was taken from AAMI for the purpose test on spectrometer. the spectrometer test were performed at Akaki Basic Metal Industry .The result indicated that the material type was unable to identify from the industry available resource of the company but the composition are identified.

**Table 3 Material composition of Chery 100hp transfer case**

Minerals	C	Si	N	P	S	Cr	Mo	Ni	Al
Percentage	1.53	1.35	0.75	0.04	>0.096	>0.071	0.00031	0.006	0.003
Minerals	Cu	Ti	V	W	Sn	As	Fe		
Percentage	0.072	>0.026	0.026	<0.04	0.013	0.016	95.9		

The transfer case material type is found that AISI Alloy steel 4140 which has the physical properties are as follows

**Table 4 Physical property of AISI Alloy steel 4140**

Physical Property of AISI Alloy steel 4140	
Density	7.85g/cm <sup>3</sup>
Melting point	1416°C
Tensile strength	655Mpa
Yield strength	415Mpa
Bulk modulus	140Mpa
Shear modulus	80Gpa
Elastic modulus	190-210Gpa
Poisson ratio	0.27-0.30
Elongation	25-75%

## 2.3. Mathematical Modeling

### 2.3.1. Physical modeling

Power transmission system of Chery 100 hp tractor works with the same manner of any other tractor or automotive system. During two wheel drive the engine power is on the two tractor wheels and the other two wheels allows to spin. The power came from the engine directly transferred in to clutch to engage the drive line and transfer the power towards the gearbox, then the power transferred in to the gear box for reduction of motor speed in to vehicle speed. Finally the differential allow the rear axle to rotate with respect to each other and power transferred in to wheel. Whenever the traction force needed to be increased the power needed to be split in to four that will give the maximum drive traction and accomplished by engaging the differential gear in to transfer case. From the transfer case

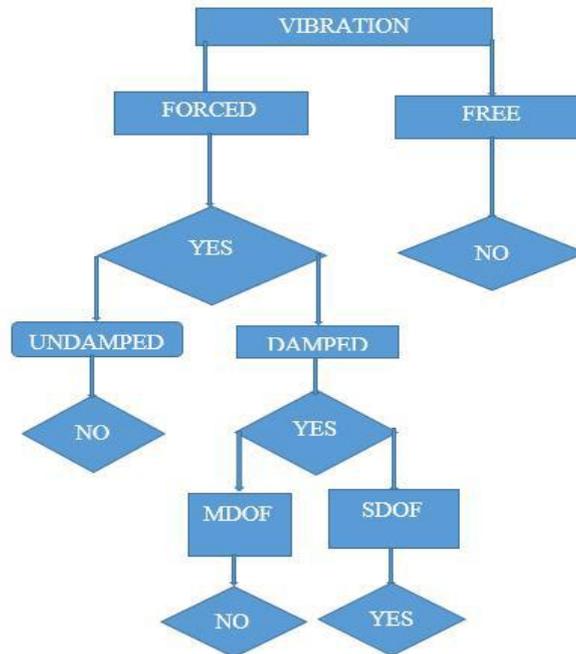
the power will be transferred into front axle to make four wheel drive. This will be shown on simple flow chart below.

➤ **General Assumptions**

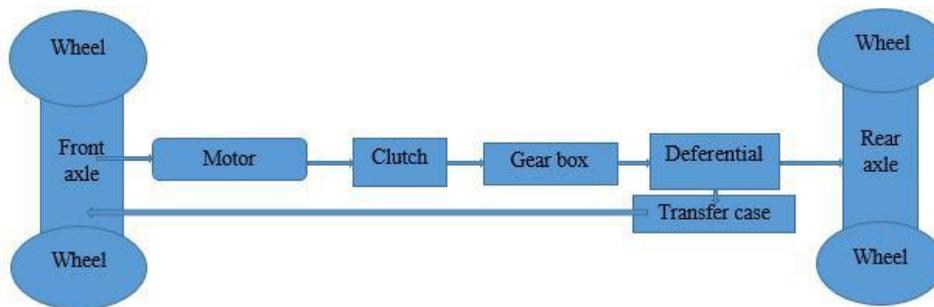
✓ Forced vibration:-The system vibrates under the influence of external force.

✓ Single degree of freedom:-The geometrical pattern of transfer case is constrained within 20 bolts and the failure happens in the axial direction.

✓ Viscous damping:-The damping system of the entire driveline and gear box and system is viscous damper of oil type SEA140. This can be easily explained on the chart below



**Figure 2 Forced vibration damped and SDOF**



**Figure 3 position of the transfer case in the tractor**

➤ **Degree of freedom**

According to geometrical pattern of transfer case the degree of freedom considered as a single degree of freedom.

**2.4. Mathematical Analysis**

A mass 18.7 kg of transfer case attached in to the differential with a fixed support constrained by 20 bolts.

Damping also provided in to the system by SEA140 gear oil which has a physical property of viscosity index 97 and density 0.91kg/l.

**Assumptions**

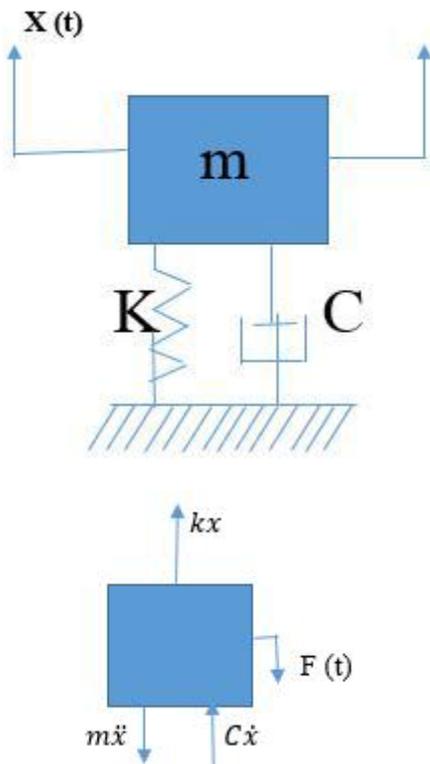
- Lumped mass a system with finite number of degree of freedom are called lumped parameter Stiffness

proportional to displacement (according to Hooke's law assume simple harmonic vibration)

- Damping proportional to velocity (Motion is oscillatory motion).

Single degree of freedom (assume the transfer case is constrained with bolts but only move on vertical axis)

- The system is considered to be mass spring damper system to get the equation of motion of natural vibration.



For the free body diagram of the above figure along y the summation of the force acting on transfer case is not equal to zero since there is forced vibration there is external excitation and the total force acting on the transfer case are damping force, inertia force excitation force and restoring force.

Gravitational force and spring force are negative because the gravitational force is downward force and the spring negative sign indicates the spring pulls when it is stretch and pushes when it is compressed. There is a viscous damper which dissipate the friction

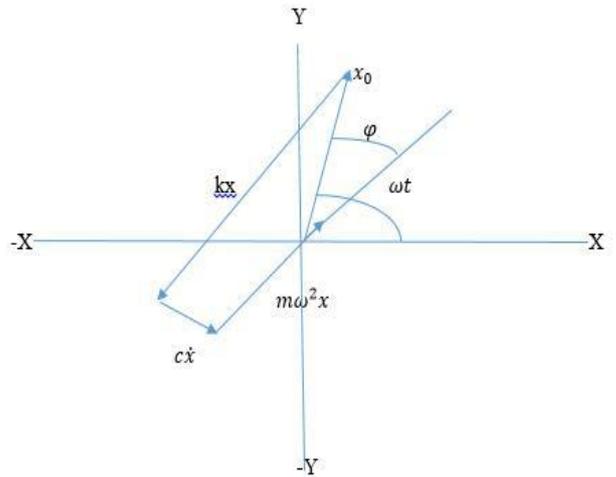
Newton's law

$$\sum F = ma \text{-----}(2.1)$$

$$\frac{d^2x}{dt^2} = m\ddot{x} - kx \text{-----}(2.2.)$$

$m\ddot{x} - kx = mg$  Which is 2nd order homogeneous equation

when adding friction dissipation force which is damping force.



$$F_D = -Cv = C\dot{x} \text{-----}(2.3)$$

The dissipation force is directly related with velocity v (kind of air dragger the faster it moves the larger the dragger force related to the speed) then importing the friction dissipating force  $F_D$  and the excitation force  $F(t)$  in to the above equation.

$$ma = -kx - c\dot{x} + m\ddot{x} + F(t) \text{---}(2.4)$$

Is the excitation force caused by harmonic vibration and represented by using sine wave equation.

$$kx + c\dot{x} - m\ddot{x} = X_0 \sin \omega t = X(t) \text{-----}(2.5)$$

The above solution has homogeneous and particular solution.

$$X(t) = X_{particular} + X_{homogeneous} \text{-----} 2.6$$

$$X_{homogeneous} = kx + c\dot{x} - m\ddot{x} = 0 \text{----}(2.7)$$

$$X = X_0 e^{-\xi\omega t} \sin(\omega_n t + \varphi) \text{-----}(2.8)$$

$$X_h = e^{-\xi\omega_n t} [A \cos\sqrt{1-\xi^2} \omega_n t + B \sin(1-\xi^2 \omega_n t)] \text{(2.9)}$$

$X_{particular}$

$$X_p = X_0 \sin(\omega t + \varphi) \text{-----} 2.9$$

Where  $\varphi$  is angle where displacement vector lag to force vector.

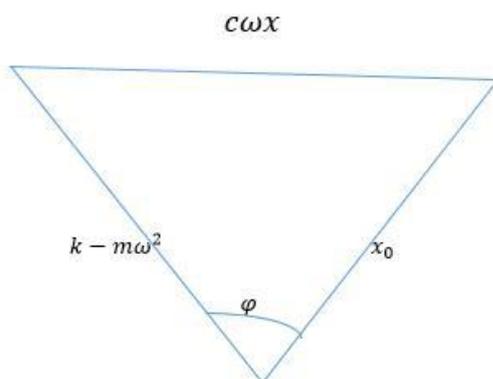
$$\dot{x}_p = \omega X_0 \sin(\omega t - \varphi + \pi/2) \text{---} 2.10$$

$$\ddot{x}_p = \omega^2 X_0 \sin(\omega t + \varphi + \pi) \text{---} 2.11$$

Substituting the above three equation in to general solution

$$-m\omega^2 X_0 \sin(\omega t - \varphi + \pi) + c\omega X_0 \sin(\omega t - \varphi + \pi/2) + kX_0 \sin(\omega t - \varphi) - F_0 \sin\omega t$$

The picture below shows the graphical representation of the above equations.



$$X_0 = \frac{F_0}{\sqrt{(k - m\omega^2)^2 + c\omega^2}} \text{----}(2.13)$$

$$\tan\varphi = \frac{c\omega x}{kx - m\omega^2} \text{-----}(2.14)$$

$$\tan\varphi = \frac{c\omega x / kx}{k(x - m\omega^2 / k)} \text{-----}(2.15)$$

$$\tan\varphi = \frac{c\omega/k}{1 - m\omega^2/k} \text{-----}(2.16)$$

$$\varphi = \tan^{-1} \frac{c\omega/k}{1 - m\omega^2/k} \text{-----} 2.17$$

$$\text{if } \frac{k}{m} = \omega_n^2 \text{ then } \frac{m}{k} = \frac{1}{\omega_n^2}, \frac{k}{m} = \omega_n^2 \text{ then, } \frac{m\omega^2}{k} = \frac{\omega^2}{\omega_n^2} \text{-----}$$

$$\frac{C\omega}{k} = \frac{C}{C_c} * \frac{C_c}{2m} * \frac{2m}{k} * \omega \text{----}(2.18)$$

$$\frac{C}{C_c} = \xi, \frac{C_c}{2m} = \frac{\omega_n m}{k} = \frac{1}{\omega_n^2} \text{----}(2.19)$$

$$\frac{c\omega}{k} = \frac{\xi 2\omega}{\omega_n} \text{-----}(2.20)$$

Substituting in to the above equations

$$X_0 = \frac{F_0/k}{\sqrt{(1 - \frac{m\omega^2}{k})^2 + \frac{c\omega^2}{k}}} \text{----} (2.21)$$

$$\frac{F_0}{k} = X_{st} \text{-----}(2.22)$$

$$X_0 = \frac{X_{st}}{\sqrt{(1 - \frac{\omega^2}{\omega_n^2})^2 + 2\xi\omega/\omega_n^2}} \text{-----} 2.23$$

$$\varphi = \tan^{-1} \frac{2\xi\omega/\omega_n}{1 - \frac{\omega^2}{\omega_n^2}} \text{-----}(2.24)$$

$$X_p = X \sin(\omega t - \varphi) \text{-----}(2.25)$$

After substituting X in the equation

$$X_p = \frac{X_{st}(\omega t - \varphi)}{\sqrt{(1 - \frac{\omega^2}{\omega_n^2})^2 + 2\xi\frac{\omega^2}{\omega_n}}} \text{-----}(2.26)$$

$$X = X_p + X_h \text{-----}(2.24)$$

Substituting particular and homogeneous solution in to the general solution

$$X = e^{\xi\omega_n t} [A \cos(\sqrt{1-\xi^2} \omega_n t + B \sin(1-\xi^2 \omega t)] + \frac{X_{st}(\omega t - \phi)}{\sqrt{(1-\frac{\omega^2}{\omega_n^2})^2 + 2\xi\frac{\omega^2}{\omega_n}}} \dots (2.25)$$

“The general solution”

$$X = e^{\xi\omega_n t} [A \cos(\sqrt{1-\xi^2} \omega_n t + B \sin(1-\xi^2 \omega t)] + \frac{X_{st}(\omega t - \phi)}{\sqrt{(1-\frac{\omega^2}{\omega_n^2})^2 + 2\xi\frac{\omega^2}{\omega_n}}} \dots 2.25$$

Where A and B are constants that will be found from initial condition.

In damped natural frequencies and the damping ratio are easily calculated from the physical parameters *m*, *k* and *c*, so it is quite easy to get an estimate of the behavior of real system.

To find the stiffness matrix *K* the modulus elasticity and the moment of inertia should be known.

The moment of inertia for the transfer case of the tractor will be difficult to calculate since it has irregular shape and have six faces. Each dimension of the transfer case is directly measured from the real transfer case during technical specification.

The summation of all moment of inertia of six face equal to the moment of inertia for the transfer case.

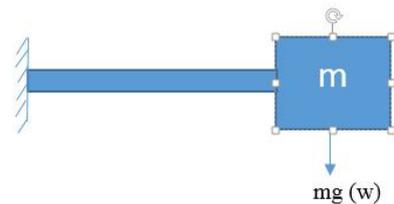
Whereas the transfer case material type measured by spectrometer from Akaki Basic metal Industry is categorized as AISI Alloy steel 4140. The Physical property

of this material indicate the value for elastic modulus is between 190 to 210Gpa.

$$I_T = \sum_1^6 I = 3.71 \cdot 10^{-4} m^4 = 3.71 \cdot 10^8 mm^4$$

*E*=190-210Gpa The real measurement the transfer case the mass and the length of the transfer case is equal to 18.7kg and 145 mm respectively. These all helps to find the value of stiffness *K*.

### 2.5 Stiffness (k)



#### ➤ Assumptions

Assuming the elementary beam theory

- The beam is loaded only in the y direction.( because the power transfer from the differential to the transfer case is in a vertical direction)
- Deflections of the beam (shaft) is small in comparison to the transfer case
- The material of the beam is linearly elastic, isotropic, and homogeneous.(...is a material property of the shaft found from catalog of the shaft)
- The system is considered to be a cantilever beam because the shaft is assumed to be mass less comparing with the transfer case and the mass is concentrated on one end which is the transfer case. From the concept of strength of material,

## III. RESULT AND DISCUSSION

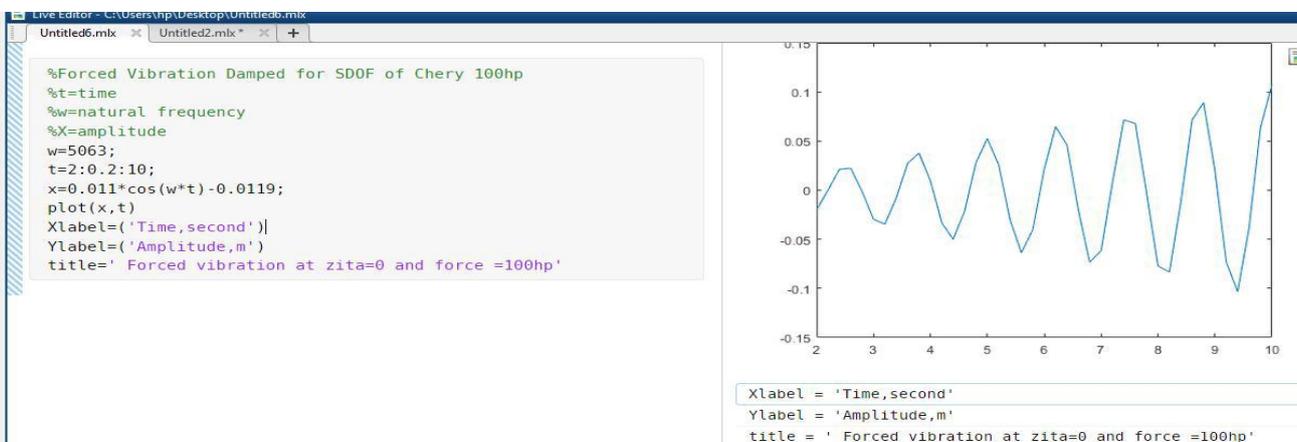


Figure 4 amplitude versus time plot at Zita =0 and 100hp

The above plot shows that as the time increase the amplitude decrease this is the clear indication of the system is not damped.

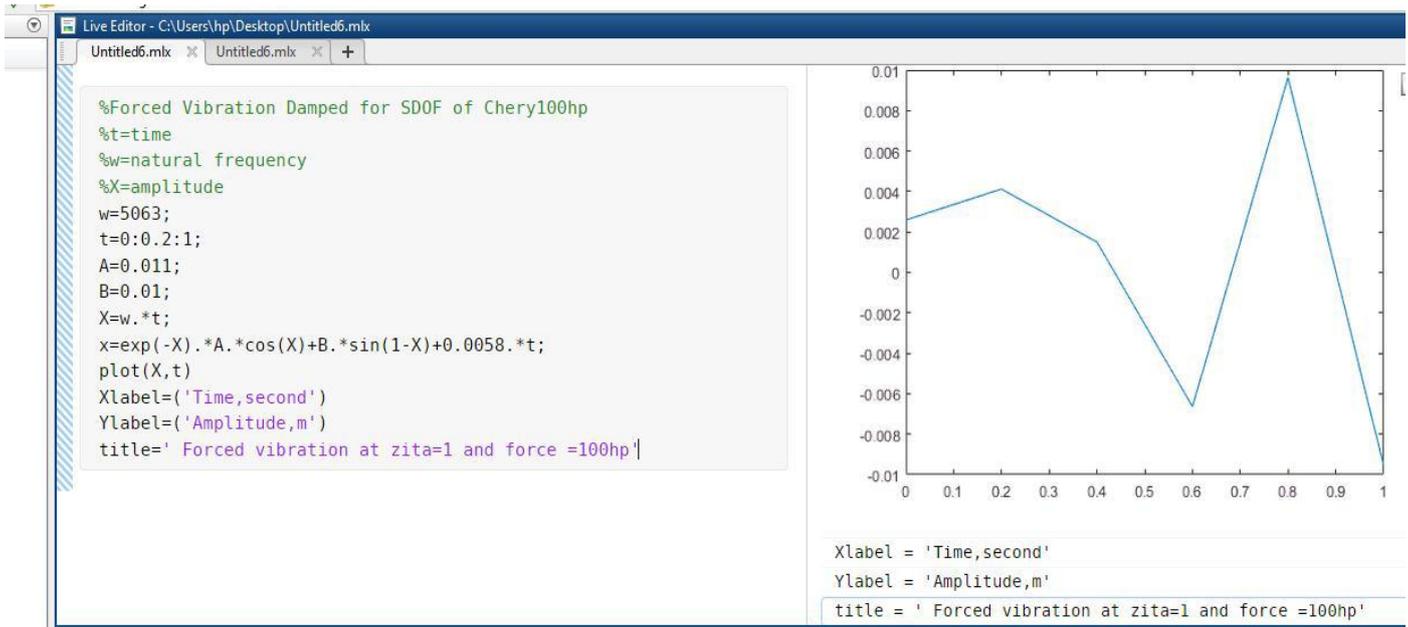


Figure 5 plot of amplitude versus time graph at Zita=1 and 100hp

The plots specifies that the increment on the damping from 0 to 1 has foundation for a system to be changed means the

system to be damped. That implies one of the failure reason for the system is damping oil type. Amplitude versus time graph at  $\xi=1$

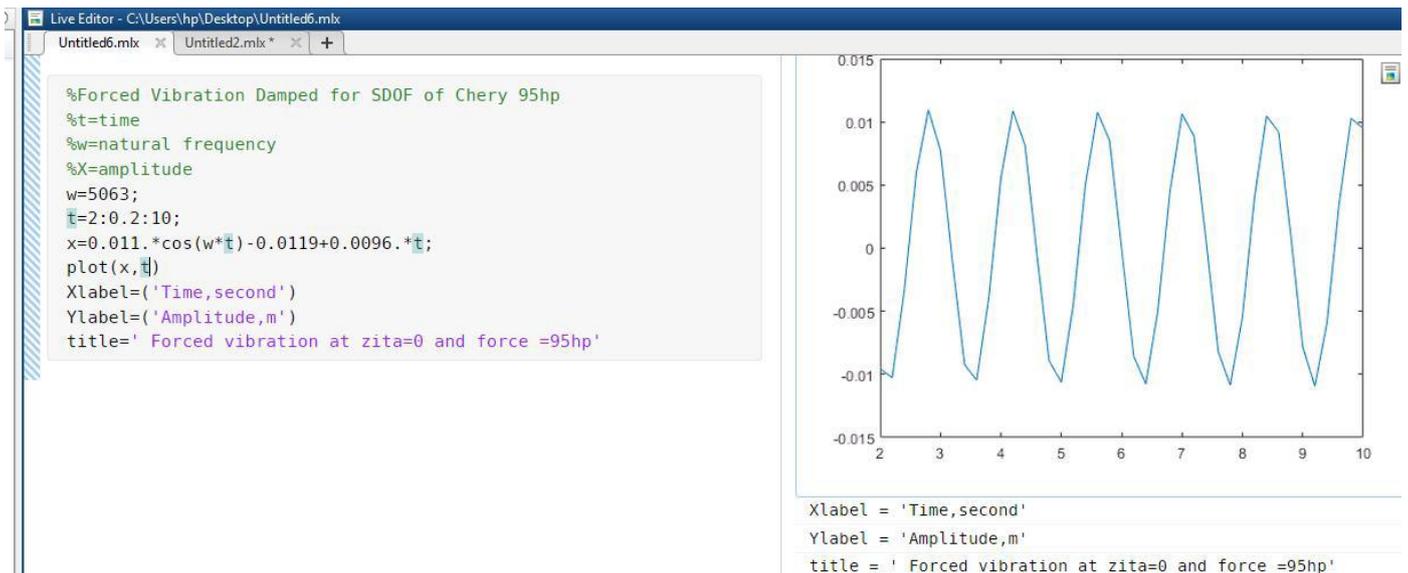


Figure 6 plot of amplitude versus time graph at zita =0 and 95hp

Taking the equation in to Matlab software and plot amplitude versus time graph at  $\xi=0$  it will give the figure below



Figure 7 plot of amplitude versus time graph at zita=1 and 95hp

**Specification of SEA140 gear box oil**

Table 2 Physical properties of SEA 140 oil specification

Viscosity@100°C	28-33
Viscosity index	90
Pour point °C	-3
Flash point °C	190
Density@15°Ckg/l	91

The damping factor for the existing oil specification is calculated and found to be zero with this damping factor the general solution is plotted for amplitude versus time graph for both 95hp and 10 hp excitations.

Which indicates that the graph the system remain undamped with this damping factor which is  $\xi=0$ . The amount of damping factor chosen to be  $\xi=1$  which is the critical damping factor of forced vibration system.

Again the amount of damping factor inserted in to the general solution to plot whether the system response is

damped or not. Based on the plot the system response showed that at the value of  $\xi=1$  the system damped. In order to proceed with this damping factor the oil specification should be chosen from available literature result.

The value of damping factor varied from one type of oil to the other type of oil this leads to know the better oil type that has good damping factor. The following gear oil specification has taken from literatures for better result of damping factor.

Table 3 Test data made for top damping gear oil

Oil type	Kv @T 100c	V <sub>Inde</sub> x	ρ@15.6c	FP	PP
SAE J306	445 mm <sup>2</sup> /s	2270	1298Kg/l	92	97
SAE90	15.5mm <sup>2</sup> /s	90	0.9084Kg/l	482	10
SAE140	27.5mm <sup>2</sup> /s	93	0.9165Kg/l	486	10
SAE250	46mm <sup>2</sup> /s	119	0.9157Kg/l	496	16
ISO 100	100 mm <sup>2</sup> /s	98	0.880*10 <sup>3</sup>	250	-24
ISO220	233 mm <sup>2</sup> /s	96	0.889 *10 <sup>3</sup>	496	-18
ISO 150	14.8mm <sup>2</sup> /s	98	0.884 *10 <sup>3</sup>	256	-24
ISO 68	8.6mm <sup>2</sup> /s	97	0.878 *10 <sup>3</sup>	242	-30
SAE 85W-140	30.3mm <sup>2</sup> /s	103	0.901 *10 <sup>3</sup>	200	-12



The value for each damping oil factor are summarize as follows

**Table 4** calculated value of Zita for different damping oil

Gear Oil type	Zita values
SAE J306	$5.29 \times 10^{-6}$
SAE90	0.027
SAE140	$5.3 \times 10^{-6}$
SAE250	$4.81 \times 10^{-6}$
ISO 100	$7.44 \times 10^{-6}$
ISO220	$5.68 \times 10^{-11}$
ISO 150	$5.35 \times 10^{-3}$
ISO 68	$4.81 \times 10^{-6}$
SAE85W-140	$7.68 \times 10^{-6}$

According to the calculated value of damping factor the better number that can neighboring to one ( $\xi=1$ ) comparing to the others is SAE 90 gear oil type for this system.

**Natural frequency values for different type of materials**

Material	Modulus of elasticity(Gpa)
AluminumAlloy1100	69
Copper Alloy	115
Graycast Iron	169
Steel Alloy	200
Carbon and low steel alloy (ASTM)	103

**Figure 8** Modulus of elasticity for different metals

Material type	Natural frequency (rad/sec)	Natural frequency (Hz)
Aluminum Alloy 1100	1347.09	214.38
Copper Alloy	2769.5	440.70
Gray cast Iron	4297.74	684.00
Steel Alloy	5063	805.80
Carbon and low steel alloy(ASTM)	3091	491.94

**Results of mathematical modeling generated from Matlab software**

The graph shows the comparison experimental analysis with mathematical analysis. The plot by mathematical modeling at 100hp and calculated value of damping factor generated

from Matlab software agree with the experimental analysis of first high gear speed zone plot.

This implies the exiting force that cause vibration is due to the first high gear speed zone and the mathematical modelling is straightforward.

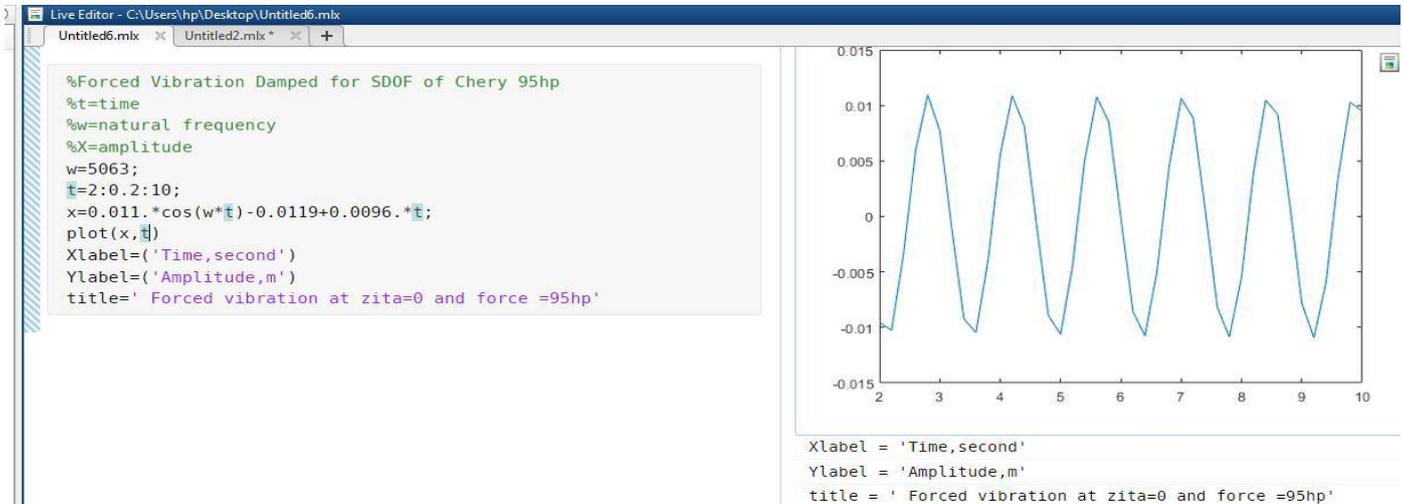
The following equation is the general solution for the system after introducing the constants A, B, natural frequency and deflection to the general solution the following graphs are plotted.

$$X = e^{\xi\omega_n t} [A \cos(\sqrt{1-\xi^2} \omega_n t) + B \sin(1-\xi^2 \omega t)] + \frac{X_{st}(\omega t - \varphi)}{\sqrt{(1-\frac{\omega^2}{\omega_n^2})^2 + 2\xi\frac{\omega}{\omega_n}}}$$

Taking the exiting force 95hp at calculated damping factor

$$\xi = 6.56 \times 10^{-16} \approx 0$$

$$X = 0.011 \cos \omega t - 0.0119 + 0.0096 t$$



**Figure 9 Forced vibration of the system at 95hp and existing damping factor**

It is observed that the system is not disturbed on the available oil damper when the excitation force is 95 hp regardless of the oil type this indicates that SAE 140 oil type is not the failure problem on the 95hp excitation force.

Taking the exiting force 95hp at critical damping factor ( $\xi=1$ )

$$X = e^{-\omega t} (0.001 \cos(\omega t) - 0.0647 \sin(1-\omega t) + 0.0053 t + 0.0018)$$

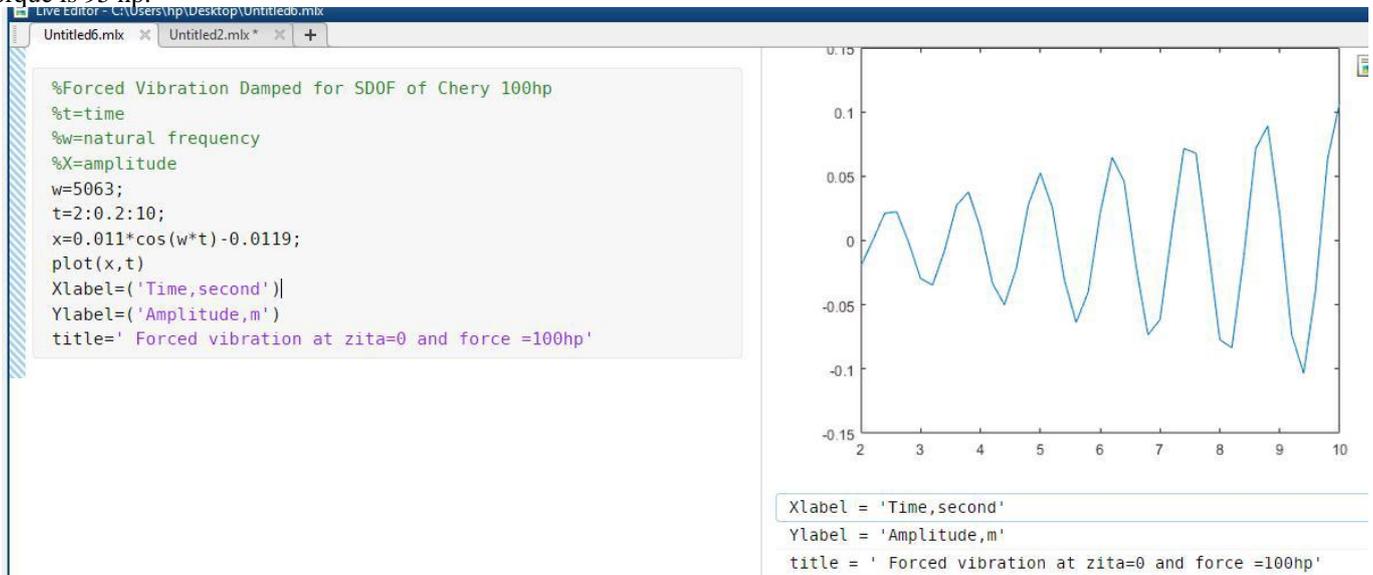


**Figure 10 Forced vibration of the system at 95hp and 1 damping factor**

The plot above indicates that the forced vibration result at the excitation torque is 95 hp but the damping factor is 1 which is critically damped. The plot specify that the system is damped and the amplitude approaches to zero at time increase which means the system is okay when the input torque is 95 hp.

Talking the exiting force or torque is 100hp at calculated damping factor  $\xi = 6.56 \cdot 10^{-16} \approx 0$

$$X = 0.011 \cos \omega t - 0.0119 + 1.9 \cdot 10^{-13} t$$

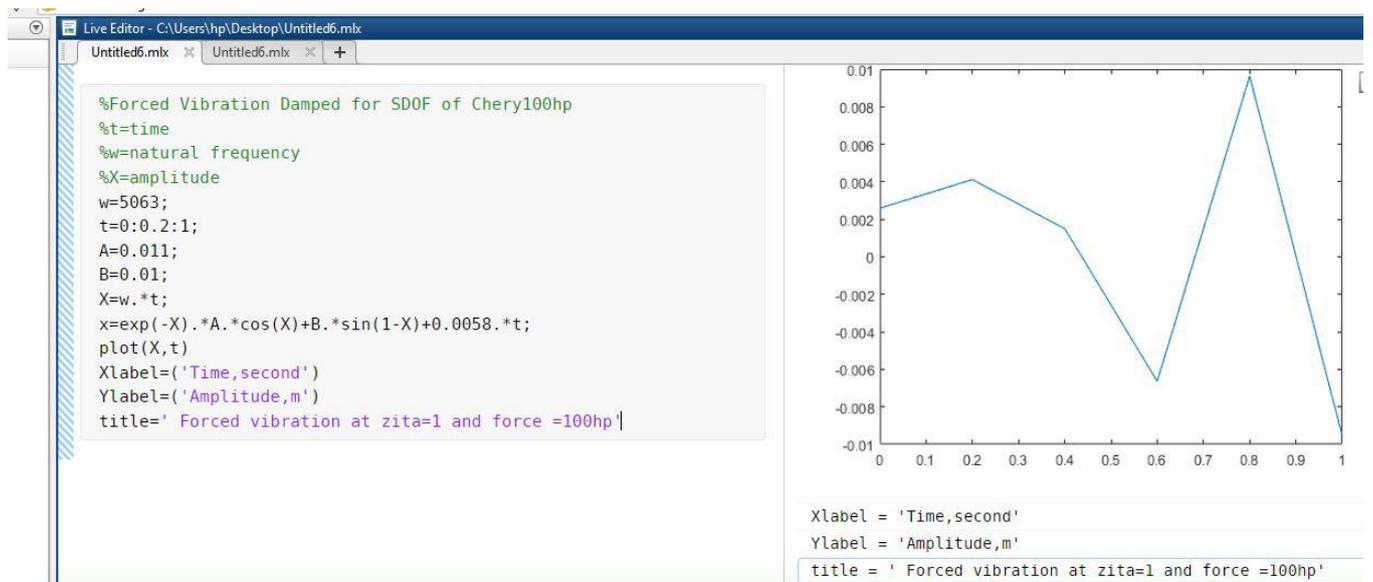


**Figure 11 Forced vibration of the system at 100hp and existing damping factor**

As it can be seen from the plot the graph looks undamped not sinusoidal and unsteady this confirms there is a clear problem on the transfer case when the amount of torque is 100hp and uses the existing damping coefficient.

Talking the exiting force 100hphp at critical damping factor ( $\xi=1$ )

$$X = e^{-2069t}(0.001 \cos(2069t) + 0.01 \sin(1-\omega t) + 0.0058t)$$



**Figure 12 Forced vibration of the system at 95hp and 1 damping factor**

The above graph shows the result from a mathematical model on the forced vibration analysis at the value of damping coefficient =1 or at a critically damping value. According to the plot the graph shows the amplitude becomes zero at time increase this indicates the system will be damped by changing oil type. According to the Investigation for the damping capacity data for damping capacity and material microstructure the result indicates that damping in steel and cast iron examined strongly and comparison has made cast and alloy steel cast iron exhibit better damping in most cases. The effect of mechanical properties of materials on natural frequency and mode shapes of heavy vehicle gearbox

transmission casing were studied for the vibration response study of casing. Grey cast iron grade FG 260, Structural Steel, Al alloy and Mg Alloy materials were analyzed for different twenty mode. According to the study the frequency Al alloys (1291-3829) band for all materials are grey cast iron (1002.5-2954.8), structural Steel (1306-3879), and Mg Alloys (1273-3784). The research work has concluded that the mechanical properties are directly related with natural frequency and vibration mode shapes of structural rigidity structural steel have excellent properties

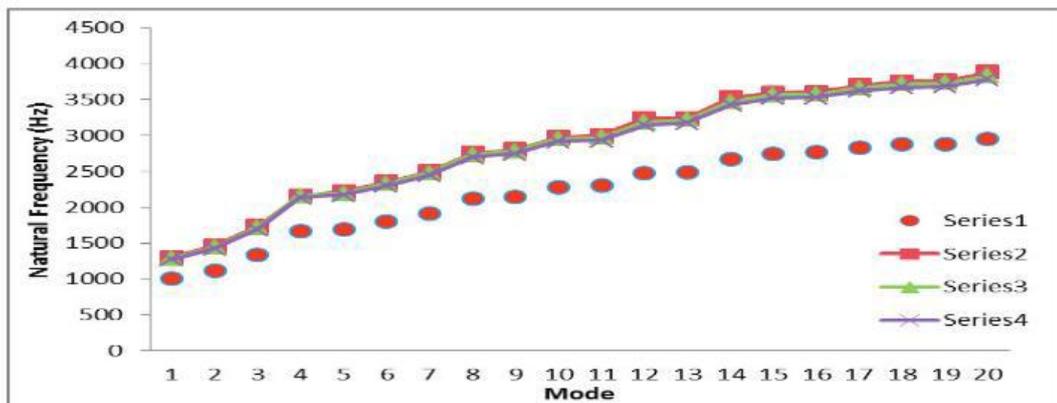


Figure 8 Natural frequency graph Grey cast Iron (1) , Structural Steel (2), Al Alloys (3), Mg Alloys (4).

**Figure 13 Material property versus natural frequency**

**REFERENCES**

[1]. Villibor GP, Santos FL, Queiroz DM De, Medina D. Acta Scientiarum Vibration levels on rear and front axles of a tractor in agricultural operations. 2014.doi:10.4025/actascitechnol.v36i1.18170

[2]. Wrachien DD. Actual tasks on agricultural engineering. 44th Int Symp. 2016;47(July 2016):1. doi:10.13140/RG.2.1.3677.8088

[3]. Mehari A, Eriksson F, Kuttikal L, Supervisor J, Linderholt A. Parametric study of a dog clutch used in a transfer case for trucks. 2013.

[4]. Alsalaet J. Vibration Analysis and Diagnostic Guide. 2016; (December). Tuma J. Gearbox noise and vibration prediction and control. Int J Acoust Vib. 2009. doi:10.1002/9780470209707.ch88

[5]. Komag CMG. Vibration analysis of mechanical systems with utilisation of GRAFSIM and CATGEN software. 2007; 23(1):3-7.

[6]. Howard I. VIBRATION SIGNAL PROCESSING USING MATLAB®.

[7]. Dhande T, Patil RB. Design Optimization and Analysis of Industrial Gearbox Casing to Reduce Wear of Gear and Bearings Due to Vibrations:

Review. Int J Sci Res ISSN (Online Index Copernicus Value Impact Factor. 2013;14(5):2319-7064.