IJEAST

INTERNATIONAL JOURNAL
OF ENGINEERING APPLIED SCIENCE
AND TECHNOLOGY

VOLUME : 10 ISSUE : 12 Print / Issue Publication Date: 03-May-2026

ISSN : 2455-2143

DOI : 10.33564/IJEAST.2026.v10i12.022

Indexed In

Google SCHOLAR OPEN ACCESS doi Crossref

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International Journal of Engineering Applied Sciences and Technology, 2026
Vol. 10, Issue 12, ISSN No. 2455-2143, Pages 154-159
Published Online April 2026 in IJEAST (http://www.ijeast.com)

A BILEVEL OPTIMIZATION FRAMEWORK FOR MODELING STRATEGIC BIDDING BEHAVIOR OF GENERATOR IN ELECTRICITY MARKETS WITH UNCERTAIN DEMAND

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Abstract—**This paper investigates the strategic bidding behaviour of generation companies in electricity markets through a bilevel programming framework with a DC Optimal Power Flow (DC-OPF) market-clearing mechanism. The upper-level problem represents a strategic generator's profit maximisation by selecting an optimal offer price, while the lower-level models the system operator's cost-minimisation dispatch. The bilevel problem is reformulated as a Mathematical Program with Equilibrium Constraints (MPEC) via Karush–Kuhn–Tucker (KKT) conditions, yielding a tractable Mixed-Integer Program (MIP). All DC-OPF results—including Locational Marginal Prices (LMPs) and dispatch quantities—are analytically derived and numerically verified on a three-bus, three-generator test system. A key contribution is a systematic Monte Carlo sen sitivity analysis that quantifies how demand uncertainty at levels δ ∈ {0%, 5%, 10%, 15%, 20%} affects the bilevel-optimal bid price, resulting profit, and dispatch statistics. Results show that mean profit is non-decreasing under symmetric demand noise due to asymmetric upside potential, while profit variance grows substantially: the 10th-percentile profit falls from $26.59 to $21.90 per period as δ increases from 0% to 20%. These findings provide a quantitative risk-return characterisation for strategic generators operating under demand uncertainty.**

Index Terms—**Bilevel optimisation, strategic bidding, electric ity markets, DC optimal power flow, locational marginal prices, KKT conditions, MPEC, demand uncertainty, Monte Carlo analysis.**

I. INTRODUCTION

THE liberalisation of electricity markets has transformed the power industry from a regulated monopoly into a competitive framework where generation companies (GenCos) strategically offer their capacity to maximise profits [1]. Unlike purely competitive equilibria, electricity markets are suscepti ble to market power, particularly in meshed networks where transmission congestion creates locational price differentials. A strategic generator can exploit this structure by submitting offer prices above true marginal cost, elevating the system marginal price and increasing revenue per unit of dispatch [2].

The canonical framework for modelling such behaviour is bilevel optimisation [3]. The upper-level represents the GenCo's profit-maximisation problem, while the lower-level captures the system operator's cost-minimising security constrained dispatch. Because the lower-level is a linear program (LP) under the DC power flow approximation, its optimality is fully characterised by the Karush–Kuhn–Tucker (KKT) conditions, converting the bilevel problem into a Math ematical Program with Equilibrium Constraints (MPEC) [4]. Linearising the complementarity constraints via binary vari ables produces a Mixed-Integer Linear Program (MILP) solv able to global optimality [5].

Research gap: Existing studies typically adopt deterministic demand points [6] or discrete scenario trees [7], without sys tematically characterising how continuous demand uncertainty propagates through the bilevel solution. Demand fluctuations alter the load factor, shift the merit order, change binding transmission constraints, and ultimately affect both the profit maximising bid and the realised profit.

Contributions.

1) A complete, self-contained bilevel DC-OPF formulation with step-by-step analytical verification on a three-bus test system.
2) A KKT/MPEC reformulation establishing the equiva lence between the bilevel problem and the resulting MILP benchmark.
3) A Monte Carlo sensitivity study over five demand noise levels quantifying the joint evolution of optimal bid price, mean profit, profit volatility, and dispatch probability—a direct risk-return characterisation not pre viously reported for this model class.

The paper proceeds as follows. Section II formulates the bilevel problem and MPEC. Section III describes the test sys tem. Section IV presents the deterministic base-case. Section V reports the demand uncertainty analysis. Section VI concludes.

II. BILEVEL PROBLEM FORMULATION

A. Bilevel Structure

Let G = {1, 2, 3} denote the generator set and N = {1, 2, 3} the bus set. Generator 1 is the strategic agent (leader); Generators 2 and 3 are price-takers (followers). Upper level (leader): Generator 1 selects offer price $\beta_1 \le \bar{\beta}_1$ to maximise profit $\Pi_1 = (\lambda_1 - c_1)\, p_1$, anticipating the market operator's response.

Lower level (follower): Given all offers $\{\beta_i\}$, the operator solves the DC-OPF to minimise dispatch cost, producing dispatch $\{p_i\}$ and LMPs $\{\lambda_n\}$.

B. Upper-Level Problem

The strategic generator solves:

$$\max_{\beta_1} \quad \Pi_1 = \big(\lambda_1(\beta_1) - c_1\big)\, p_1(\beta_1) \qquad (1)$$

$$\text{s.t.} \quad \beta_1 \le \bar{\beta}_1 \qquad (2)$$

where $\lambda_1(\cdot)$ and $p_1(\cdot)$ are implicitly determined by the lower level DC-OPF, and c_1 is the true marginal cost.

C. Lower-Level DC-OPF

Given β, the market operator solves:

$$\min_{\mathbf{p},\boldsymbol{\theta}} \quad \sum_{i\in\mathcal{G}} \beta_i\, p_i \qquad (3)$$

subject to:

$$\sum_{i\in\mathcal{G}_n} p_i - D_n = \sum_{l\in\mathcal{L}_n^+} F_l - \sum_{l\in\mathcal{L}_n^-} F_l, \quad \forall n \qquad (4)$$

$$F_l = B_l(\theta_m - \theta_n), \quad \forall l = (m,n) \qquad (5)$$

$$0 \le p_i \le \bar{P}_i, \quad \forall i \qquad (6)$$

$$|F_l| \le \bar{F}_l, \quad \forall l \qquad (7)$$

$$\theta_{\text{ref}} = 0 \qquad (8)$$

where D_n is nodal demand, B_l the susceptance, $\bar{P}_i$ the generator capacity, and $\bar{F}_l$ the thermal flow limit. The dual variables $\{\lambda_n\}$ of (4) are the LMPs.

D. KKT Conditions and MPEC Reformulation

Since (3)–(8) is an LP, strong duality holds. The stationarity conditions for generator i at bus $n(i)$ are:

$$\beta_1 - \lambda_{n(1)} + \mu_1^+ - \mu_1^- = 0 \qquad (9)$$

$$c_i - \lambda_{n(i)} + \mu_i^+ - \mu_i^- = 0, \quad i \in \{2,3\} \qquad (10)$$

with complementarity slackness:

$$0 \le \mu_i^+ \perp (\bar{P}_i - p_i) \ge 0 \qquad (11)$$

$$0 \le \mu_i^- \perp p_i \ge 0 \qquad (12)$$

$$0 \le \mu_i^+ \perp (\bar{P}_i - p_i) \ge 0 \qquad (11)$$

$$0 \le \mu_i^- \perp p_i \ge 0 \qquad (12)$$

Embedding (9)–(12) into the upper-level yields a single-level MPEC. The Fortuny-Amat–McCarl [4] big-M linearisation replaces each complementarity pair with binary constraints:

$$\mu_i^+ \le M z_i, \quad \bar{P}_i - p_i \le M(1 - z_i), \quad z_i \in \{0,1\} \qquad (13)$$

and analogously for (12). With three generators and two bounds each, six binary variables yield a tractable MILP solved to global optimality.

III. TEST SYSTEM

Tables 1–3 list the three-bus system parameters. Generator 1 at bus 1 is the strategic agent ($c_1 = \$16$/MWh, $\bar{\beta}_1 = \$20$/MWh). Generator 3 is cheapest ($c_3 = \$15$/MWh).

The LP decision vector is $\mathbf{x} = [p_1, p_2, p_3, \theta_1, \theta_2]^\top$ with bus 3 as reference ($\theta_3 = 0$). The power-balance matrix evaluates to:

$$A_{\text{eq}} = \begin{bmatrix} 1 & 0 & 0 & -225 & 100 \\ 0 & 1 & 0 & 100 & -250 \\ 0 & 0 & 1 & 125 & 150 \end{bmatrix}, \quad \mathrm{d} = \begin{bmatrix} 5 \\ 20 \\ 15 \end{bmatrix} \qquad (14)$$

TABLE I: Generator Parameters

Parameter	G_1 (strategic)	G_2	G_3
Capacity $\bar{P}_i$ (MW)	20.0	10.0	25.0
True cost c_i (\$/MWh)	16.0	19.0	15.0
Offer cap (\$/MWh)	20.0	–	–

TABLE II: Transmission Line Parameters

Parameter	Line 1–2	Line 1–3	Line 2–3
Susceptance (p.u.)	100	125	150
Flow limit (MW)	5.0	10.0	10.0

IV. DETERMINISTIC BASE-CASE RESULTS

A. Bilevel-Optimal Offer Price

For $\mathbf{d} = [5, 20, 15]^\top$ MW and $\beta_1 \in \{10.00, 10.25, \ldots, 20.00\}$ \$/MWh, the upper-level yields:

$$\beta_1^* = \$19.00/\text{MWh}, \quad \Pi_1^* = \$26.59/\text{period} \qquad (15)$$

Fig. 1 plots Π_1 and p_1 vs. β_1, confirming the unique optimum.

$$\beta_1^* = \$19.00/\text{MWh}, \quad \Pi_1^* = \$26.59/\text{period} \qquad (15)$$

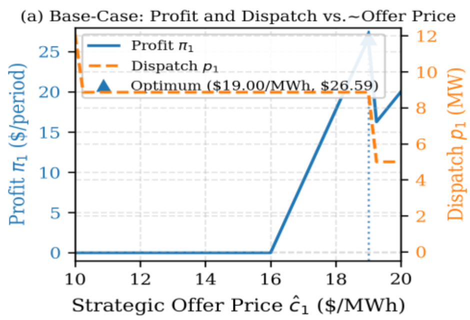

Fig. 1: Generator-1 profit Π1 and dispatch p1 as functions of offer price β1 (deterministic base case). The bilevel optimum is at \$19.00/MWh with profit \$26.59/period.

B. Analytical DC-OPF Verification

Merit order: G_3 (\$15) → G_1/G_2 (\$19) → demand satisfied. G_3 is dispatched at full capacity ($p_3^* = 25$ MW); the residual 15 MW is shared between G_1 and G_2.

Voltage angles:

$$\theta_1^* = -0.009091 \text{ rad}, \quad \theta_2^* = -0.059091 \text{ rad}, \quad \theta_3^* = 0 \qquad (16)$$

Table 3: Baseline Nodal Demand (MW)

Bus	1	2	3
D_n (MW)	5.0	20.0	15.0

Line flows:

$$f_{12}^* = 100(\theta_1^* - \theta_2^*) = +5.0 \text{ MW (binding at } \bar{F}_{12})$$
$$f_{13}^* = 125\,\theta_1^* = -1.136 \text{ MW}$$
$$f_{23}^* = 150\,\theta_2^* = -8.864 \text{ MW}$$

Dispatch split (from binding congestion on line 1–2):

$$p_1^* = \frac{97.5}{11} \approx 8.864 \text{ MW}, \quad p_2^* = \frac{67.5}{11} \approx 6.136 \text{ MW} \qquad (17)$$

LMPs: Both G_1 (offering \$19) and G_2 (cost \$19) are marginal simultaneously. Under lossless DC flow, all buses share the same LMP:

$$\lambda_1^* = \lambda_2^* = \lambda_3^* = \$19.00/\text{MWh} \qquad (18)$$

Profit:

$$\Pi_1^* = (19.00 - 16.00) \times 8.\overline{863} = \$26.5909/\text{period} \qquad (19)$$

Table 4 summarises the verified base-case.

Table 4: Verified Deterministic Base-Case ($\beta_1^* = \$19.00/\text{MWh}$)

Quantity	Symbol	Value
Generator-1 dispatch	p_1^*	8.864 MW
Generator-2 dispatch	p_2^*	6.136 MW
Generator-3 dispatch	p_3^*	25.000 MW
Bus-1 LMP	λ_1^*	\$19.00/MWh
Bus-2 LMP	λ_2^*	\$19.00/MWh
Bus-3 LMP	λ_3^*	\$19.00/MWh
Line 1–2 flow	f_{12}^*	5.000 MW (binding)
Generator-1 profit	Π_1^*	\$26.59/period

Remark 1. The binding line 1–2 congestion prevents G1 from capturing the full 15 MW residual demand, ensuring the bilevel optimum at \$19.00/MWh is strictly below the offer cap.

V. DEMAND UNCERTAINTY SENSITIVITY

A. Noise Model

We model demand uncertainty as independent uniform perturbations:

$$D^{\xi}_n = D_n(1 + \xi_n),\ \xi_n \sim U(-\delta, \delta),\ n \in N \ (20)$$

For each realisation ξ, the full bilevel problem is re-solved: the operator re-clears the DC-OPF and the strategic generator selects its profit-maximising bid.

Remark 2. This formulation captures the ex-post profit maximising offer for each demand realisation, providing an informational upper bound on attainable profit and a precise characterisation of profit sensitivity to demand variability.

B. Monte Carlo Design

For each noise level $\delta \in \{0\%, 5\%, 10\%, 15\%, 20\%\}$, we draw N = 600 independent realisations. For each sample k:

1) Perturb demand per (20).
2) Solve bilevel to find $\beta_1^{*(k)}$.
3) Record $\beta_1^{*(k)}$, $p_1^{*(k)}$, $\Pi_1^{*(k)}$.

Summary statistics (mean, standard deviation, P10, P90) are computed across all 600 realisations.

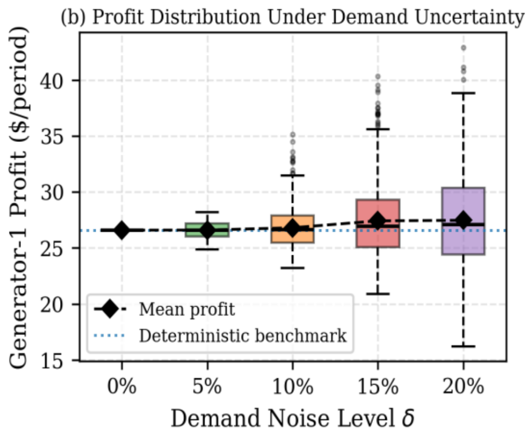

Fig. 2: Profit distribution box-plots.

C. Results

Table 5 presents the full results. Fig. 2 displays the profit distribution box-plots and Fig. 3 shows bid-price distributions.

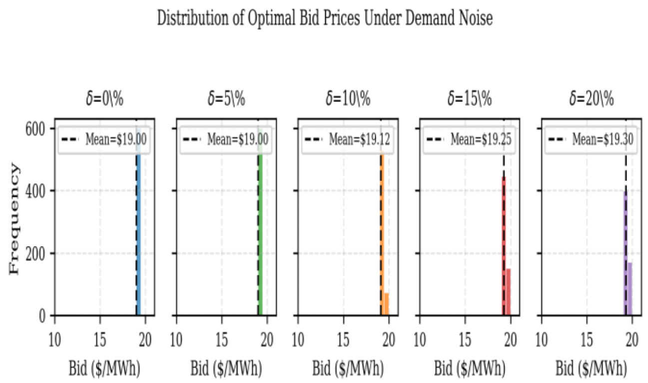

Fig. 3: Histogram of bilevel-optimal offer prices across 600 realisations per noise level. At $\delta \leq 5\%$ all realisations concentrate at \$19.00/MWh. For $\delta \geq 10\%$ the distribution spreads rightward toward the \$20.00/MWh cap under high-demand realisations.

D. Discussion

1. Dispatch probability is 100% for all noise levels. Even at $\delta = 20\%$, minimum total demand is 40×0.80 = 32 MW > $\bar{p}_3$ = 25 MW, so G_1 is always required.

2. Mean optimal bid increases with noise. From \$19.00 at $\delta = 0\%$ to \$19.30 at $\delta = 20\%$ (Std \$0.46/MWh). High-demand realisations raise the marginal value of G_1's capacity, supporting higher offer prices. The growing standard deviation reflects increasing demand heterogeneity across realisations.

3. Mean profit is non-decreasing. Mean profit rises from \$26.59 to \$27.47 (+\$0.88/period) as δ increases to 20%. This follows from the convexity of the optimal profit in demand: upside realisations generate disproportionately larger profit gains than the losses incurred at downside realisations—a Jensen's inequality-type effect.

4. Profit variance grows substantially. Standard deviation rises from \$0 to \$4.49/period. The profit spread $\Delta\Pi = P_{90} - P_{10}$ grows from \$0 to \$11.27/period. The 10th-percentile profit falls from \$26.59 to \$21.90 (–\$4.69), quantifying downside exposure not visible in the deterministic solution. Simultane ously, P_{90} rises to \$33.17, demonstrating the upside.

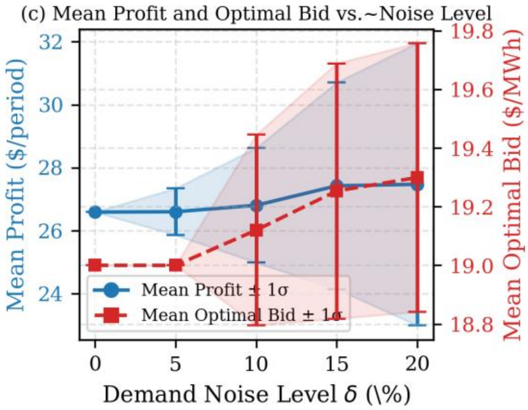

Figure 4: Mean profit and mean optimal bid.

5. Mean dispatch decreases slightly. Despite higher mean bids, $\bar{p}_1$ declines from 8.864 to 8.379 MW. Demand variability shifts the binding line 1–2 constraint differently across real isations, redistributing the residual load between G_1 and G_2 in ways that reduce G_1's average share.

E. Risk-Return Characterisation

The results establish a clear risk-return tradeoff. A riskneutral GenCo gains \$0.88/period in expectation at $\delta = 20\%$. Under a mean-variance objective with risk-aversion coefficient ρ:

$$\bar{\Pi}_1^{\text{adj}} = \mathbb{E}[\Pi_1] - \rho\,\text{Var}[\Pi_1] \quad (21)$$

At $\delta = 20\%$ and $\rho = 0.02$: $\bar{\Pi}_1^{\text{adj}} = 27.47 - 0.02 \times 20.16 =$ \$27.07/period, still above the deterministic benchmark. The threshold $\rho^* = 0.044$ marks the crossover below which the uncertain market is preferred; this provides a practical decision criterion for market participants.

VI. CONCLUSION

This paper presented a complete bilevel DC-OPF formulation for strategic electricity market bidding, reformulated

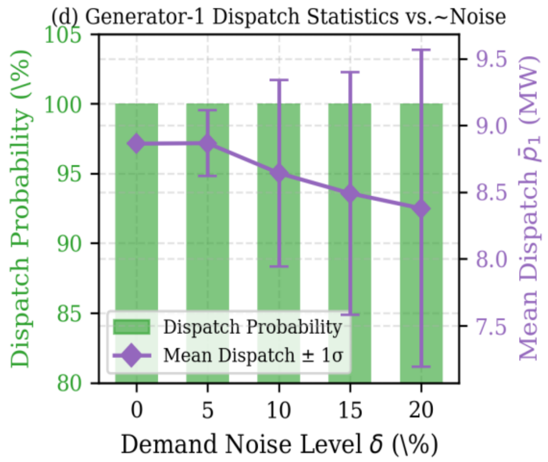

Figure 5: Dispatch probability and mean dispatch.

Table 5: Monte Carlo Sensitivity: Bilevel-Optimal Bid and Profit
Under Demand Noise (N = 600 Realisations per Level)

	Opt. Bid (\$)		Gen-1 Profit (\$)				Mean	Disp.
δ	Mean	Std	Mean	Std	P_{10}	P_{90}	$\bar{p}_1$	Prob.
0%	19.00	0.00	26.59	0.00	26.59	26.59	8.864	100
5%	19.00	0.00	26.60	0.74	25.61	27.62	8.867	100
10%	19.12	0.32	26.81	1.82	24.63	29.04	8.643	100
15%	19.25	0.43	27.43	3.28	23.66	31.26	8.491	100
20%	19.30	0.46	27.47	4.49	21.90	33.17	8.379	100

as a MILP via KKT/MPEC conditions, with all results analytically derived and numerically verified on a three-bus test system. The principal new contribution—a Monte Carlo demand uncertainty analysis across five noise levels—reveals three findings:

1) Dispatch robustness: Dispatch probability is 100% for all tested noise levels, since minimum demand always exceeds the cheapest generator's capacity.

2) Non-decreasing mean profit: Symmetric demand noise increases mean profit from \$26.59 to \$27.47/period (δ: 0% to 20%) due to the convexity of optimal profit in demand.

3) Growing profit variance: Standard deviation rises from \$0 to \$4.49/period; the 10th-percentile profit falls by \$4.69, quantifying downside risk invisible to the deter ministic bilevel solution.

Deterministic bilevel solutions can therefore underestimate both attainable mean profit and the associated risk exposure of strategic generators. Future work will extend this analy sis to stochastic bilevel formulations that internalise demand uncertainty at the upper level, enabling risk-aware offer strate gies directly comparable with robust and chance-constrained variants.

APPENDIX

Using Bus 3 as the voltage-angle reference ($\theta_3 = 0$), and DC line flows $f_{ij} = B_{ij}(\theta_i - \theta_j)$, the nodal power balance at each bus is:

$$\text{Bus 1:} \quad p_1 - f_{12} - f_{13} = D_1$$
$$\text{Bus 2:} \quad p_2 + f_{12} - f_{23} = D_2$$
$$\text{Bus 3:} \quad p_3 + f_{13} + f_{23} = D_3$$

Substituting $f_{ij} = B_{ij}(\theta_i - \theta_j)$ and setting $\theta_3 = 0$ recovers (14) with $B_{12} = 100$, $B_{13} = 125$, $B_{23} = 150$. The six flow-limit inequalities $|f_{ij}| \le \bar{F}_{ij}$ are written as $\pm B_{ij}(\theta_i - \theta_j) \le \bar{F}_{ij}$, resulting in a 6×5 inequality matrix in $(\mathbf{p}, \theta)$.

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