IJEAST

INTERNATIONAL JOURNAL OF ENGINEERING APPLIED SCIENCE AND TECHNOLOGY

VOLUME : 9 ISSUE : 06 Print / Issue Publication Date: 27-Jan-2025

ISSN : 2455-2143

DOI : 10.33564/IJEAST.2024.v09i06.001

WWW.IJEAST.COM

editor@ijeast.com

International Journal of Engineering Applied Sciences and Technology, 2024
Vol. 9, Issue 06, ISSN No. 2455-2143, Pages 01-04
Published Online October 2024 in IJEAST (http://www.ijeast.com)

PARTIALLY ANALYTICAL SOLUTION OF UNSTEADY IONIZED FLUID FLOW WITH INDUCED MAGNETIC FIELD

Md Labib Sazal
Department of Mathematics
Khulna University, Khulna, Bangladesh

Dr. Md. Mahmud Alam
Professor, Department of Mathematics
Khulna University, Khulna, Bangladesh

Abstract— **Analytical studies have been conducted on combined heat and mass transfer in unsteady ionized fluid flow passing through a vertically oscillating electrically non-conducting plate under an induced magnetic field. Through the application of similarity transformations to coupled ordinary differential equations, an analytical solution for velocity fields, induced magnetic fields, and temperature distribution is obtained. The resulting graphs illustrate variations in the obtained results with different parameter values, highlighting the effects of ionized fluid flow under the influence of the induced magnetic field**.

Keywords— **Unsteady ionized fluid flow, Induced magnetic field, Similarity transformation.**

I. INTRODUCTION

Analytically, the use of similarity transformations is a common approach to approximate the analytical solution to coupled differential equations. Nonlinear partial differential equations can pose challenges in solving; however, through similarity transformations, these equations can be converted into ordinary coupled differential equations, which are more tractable. **Monika et al. [1]** emphasized the significance of radiative-free convective flow in various industrial and environmental processes, including heating and cooling chambers, fossil fuel combustion, evaporation from large open water reservoirs, astrophysical flows, solar power technology, and space vehicle re-entry. In manufacturing industries, radiative heat transfer plays a critical role in equipment design reliability, impacting applications such as nuclear power plants, gas turbines, aircraft, missiles, and satellite devices. **Tanvir et al. [2]** highlighted the importance of considering thermal radiation effects in magnetohydrodynamic boundary layer flow across several industrial, scientific, and engineering fields. **G.V. Ramana Reddy et al. [4]** noted the interest in studying free convection flows for incompressible viscous fluids past inclined porous surfaces due to their relevance to various engineering problems such as nuclear reactor cooling, aerodynamic boundary layer control, crystal growth, food processing, and cooling towers.

II. MATHEMATICAL MODEL

Under the electromagnetic Boussinesq approximation, The MHD unsteady flow with induced magnetic field and heat and mass transfer is governed by the following equations are given by:

$$\frac{\partial u}{\partial t} - v_0 \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + 2w\Omega + g\beta_T (T - T_\infty) + \frac{\mu_e \lambda H_0}{4\pi\rho} \frac{\partial H_x}{\partial y},$$
$$\frac{\partial w}{\partial t} - v_0 \frac{\partial w}{\partial y} = \nu \frac{\partial^2 w}{\partial y^2} - 2u\Omega + \frac{\mu_e \lambda H_0}{4\pi\rho} \frac{\partial H_z}{\partial y}$$

$$\frac{\partial H_x}{\partial t} - v_0 \frac{\partial H_x}{\partial y} = \eta \frac{\partial^2 H_x}{\partial y^2} + \lambda H_0 \frac{\partial u}{\partial y} - \lambda \beta_e \eta \frac{\partial^2 H_z}{\partial y^2},$$
$$\frac{\partial H_z}{\partial t} - v_0 \frac{\partial H_z}{\partial y} = \eta \frac{\partial^2 H_z}{\partial y^2} + \lambda H_0 \frac{\partial w}{\partial y} + \lambda \beta_e \eta \frac{\partial^2 H_x}{\partial y^2}$$

$$\frac{\partial T}{\partial t} - v_0 \frac{\partial T}{\partial y} = \frac{\kappa}{\rho c_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} + \frac{Q}{\rho c_p} (T - T_\infty)$$

The corresponding boundary conditions are.

$$u = U_\infty [1 + \varepsilon (e^{i\omega t} + e^{-i\omega t})], w = 0, T = T_w, H = H_z = 0 \text{ at } y = 0$$
$$u(y,t) = 0, w = 0, T(y,t) \to T_\infty, H_x = H_z = 0 \text{ as } y$$

Where u, v and w are the x, y and z components of velocity vector, ν is the kinematic coefficient viscosity. The radiative heat flux q_r is described by the Rosseland approximation $q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y}$, σ^* and k^* are the Stefan-Boltzman constant and the mean absorption coefficient, respectively. We assume that the temperature difference within the flow is sufficiently small so that the T^4 can be expressed as a linear function after using the Taylor series to expand T^4 about the free stream temperature and neglecting higher-order terms. This results in the following approximation: $T^4 \approx 4TT_\infty{}^3 - 3T_\infty{}^4$.

III. SIMILARITY TRANSFORMATION

To solve the system of equations analytically we need to transform the governing equations into non-dimensional form, the usual non-dimensional variables are introduced below.

$$U = \frac{u}{U_\infty}, W = \frac{w}{U_\infty}, H_x = \sqrt{\frac{\rho}{\mu_e}} H_1 U_\infty, H_z = \sqrt{\frac{\rho}{\mu_e}} H_3 U_\infty, \tau = \frac{tU^2}{\nu},$$
$$\theta = \frac{T - T_\infty}{T_w - T_\infty}$$

Using all those non-dimensional variables the non-dimensional system of the coupled equation becomes

$$\frac{\partial U}{\partial \tau} - S\frac{\partial U}{\partial Y} = \frac{\partial^2 U}{\partial Y^2} + 2R'W + G_r\theta + \lambda M\frac{\partial H_1}{\partial Y}$$
$$\frac{\partial W}{\partial \tau} - S\frac{\partial W}{\partial Y} = \frac{\partial^2 W}{\partial Y^2} - 2R'U + \lambda M\frac{\partial H_3}{\partial Y},$$
$$\frac{\partial H_1}{\partial \tau} - S\frac{\partial H_1}{\partial Y} = \frac{\partial^2 H_1}{\partial Y^2}\frac{1}{P_m} - \frac{\lambda\beta_e}{P_m}\frac{\partial^2 H_3}{\partial Y^2} + \lambda M\frac{\partial f}{\partial Y}$$
$$\frac{\partial H_3}{\partial \tau} - S\frac{\partial H_3}{\partial Y} = \frac{\partial^2 H_3}{\partial Y^2}\frac{1}{P_m} + \frac{\lambda\beta_e}{P_m}\frac{\partial^2 H_1}{\partial Y^2} + \lambda M\frac{\partial g}{\partial Y},$$
$$\frac{\partial \theta}{\partial \tau} - S\frac{\partial \theta}{\partial Y} = \frac{\partial^2 \theta}{\partial Y^2}\left(\frac{1+R}{P_r}\right) + \beta\theta$$

The corresponding boundary conditions,

$Q(Y,\tau) = 1 + \varepsilon(e^{i\omega\tau} + e^{-i\omega\tau}), \xi(Y,\tau) = 1, \theta(Y,\tau) = 1$ at Y=0
$U(Y,\tau) = 0, W = 0, H_1 \to 0, H_3 \to 0, \theta(Y,\tau) \to 0$, at $Y \to \infty$

Where, τ represents the dimensionless time, Y is the dimensionless Cartesian coordinates. U and W are the velocity component in X and Y direction. H_1 and H_3 are the magnetic induction component in X and Y direction. θ is the dimensionless temperature.

$S = \frac{v_0}{U_\infty}$ (Suction Parameter), $G_r = \frac{g\beta_T(T - T_\infty)\upsilon}{U_\infty{}^2}$ (Grashof Number), $R' = \frac{\Omega_\upsilon}{U_\infty}$ (Rotational Parameter), $P_m = 4\pi\sigma\upsilon\mu_e$ (Magnetic Diffusivity Number), $M = \frac{1}{4\pi}\frac{H_0}{U_\infty}\sqrt{\frac{\mu_e}{\rho}}$ (Magnetic Parameter), $R = \frac{16\sigma^* T_\infty{}^3}{3k^*\kappa}$ (Radiation Parameter), $\beta = \frac{Q\upsilon(T_W - T_\infty)}{\rho c_p U_\infty{}^2}$ (Heat Generation or Absorption Parameter), $P_r = \frac{\rho c_p \upsilon}{\kappa}$ (Prandtl Number)

Let, $Q = U + iW$ and $\xi = H_1 + iH_3$

Then the system of coupled ordinary differential equations

$$\frac{\partial Q}{\partial \tau} - \frac{\partial^2 Q}{\partial Y^2} - S\frac{\partial Q}{\partial Y} + 2iR'Q - G_r\theta - \lambda M\frac{\partial \xi}{\partial Y} = 0$$
$$\frac{\partial \xi}{\partial \tau} - \frac{1}{P_m}\frac{\partial^2 \xi}{\partial Y^2} - S\frac{\partial \xi}{\partial Y} - i\frac{\lambda\beta_e}{P_m}\frac{\partial^2 \xi}{\partial Y^2} - \lambda M\frac{\partial Q}{\partial Y} = 0$$
$$\frac{\partial \theta}{\partial \tau} - \left(\frac{1+R}{P_r}\right)\frac{\partial^2 \theta}{\partial Y^2} - S\frac{\partial \theta}{\partial Y} - \beta\theta = 0$$

The corresponding boundary condition for the problem
$Q(Y,\tau) = 1 + \varepsilon(e^{i\omega\tau} + e^{-i\omega\tau}), \xi(Y,\tau) = 1, \theta(Y,\tau) = 1$ at Y=0
$Q(Y,\tau) = 0, \xi(Y,\tau) \to 0, \theta(Y,\tau) \to 0$, as $Y \to \infty$

IV. SOLUTION

To solve the system of coupled ordinary differential equations with the boundary condition, these equations are expressed as

$$Q(Y,\tau) = Q_0(Y) + \varepsilon(e^{i\omega\tau} + e^{-i\omega\tau})Q_1(Y)$$
$$\xi(Y,\tau) = \xi_0(Y) + \varepsilon(e^{i\omega\tau} + e^{-i\omega\tau})\xi_1(Y)$$
$$\theta(Y,\tau) = \theta_0(Y) + \varepsilon(e^{i\omega\tau} + e^{-i\omega\tau})\theta_1(Y)$$

Putting the value in the system of coupled ordinary differential equations.

$$\frac{\partial^2 Q_0}{\partial Y^2} + S\frac{\partial Q_0}{\partial Y} - 2iR'^{Q_0} - G_r\theta_0 - \lambda M\frac{\partial \xi_0}{\partial Y} = 0,$$
$$\frac{\partial^2 Q_1}{\partial Y^2} - S\frac{\partial Q_1}{\partial Y} + 2iR'Q_1 + G_r\theta_1 + \lambda M\frac{\partial \xi_1}{\partial Y} + \omega Q_1 \tan(\omega\tau) = 0$$
$$\frac{\partial^2 \xi_0}{\partial Y^2}\left(\frac{1}{P_m} + i\frac{\lambda\beta_e}{P_m}\right) + S\frac{\partial \xi_0}{\partial Y} + \lambda M\frac{\partial Q_0}{\partial Y} = 0,$$
$$\frac{\partial^2 \xi_1}{\partial Y^2}\left(\frac{1}{P_m} + i\frac{\lambda\beta_e}{P_m}\right) + S\frac{\partial \xi_1}{\partial Y} + \lambda M\frac{\partial Q_1}{\partial Y} + \omega\xi_1 \tan(\omega\tau) = 0$$
$$\left(\frac{1+R}{P_r}\right)\frac{\partial^2 \theta_0}{\partial Y^2} + S\frac{\partial \theta_0}{\partial Y} + \beta\theta_0 = 0,$$
$$\left(\frac{1+R}{P_r}\right)\frac{\partial^2 \theta_1}{\partial Y^2} + S\frac{\partial \theta_1}{\partial Y} + \beta\theta_1 + \omega\tan(\omega\tau)\theta_1 = 0$$

Then the corresponding boundary conditions becomes

$Q_0 = 1, \xi_0 = 1, \theta_0 = 1$ at $Y = 0$
$Q_0 = 0, \xi_0 \to 0, \theta_0 \to 0$ at $Y \to \infty$
$Q_1 = 1, \xi_1 = 1, \theta_1 = 1$ at $Y = 0$
$Q_1 = 0, \xi_1 \to 0, \theta_1 \to 0$ at $Y \to \infty$

Soling the equations by using boundary conditions, we obtain

$$Q_0 = \left(1 - \frac{G_r}{A_1{}^2 + SA_1 - 0.4}\right)e^{-A_5 Y} + \frac{G_r e^{-A_1 Y}}{A_1{}^2 + SA_1 - 0.4}$$
$$Q_1 = \left(1 - \frac{G_r}{A_2{}^2 + SA_2 - 0.4 + \omega\tan(\omega\tau)}\right)e^{-A_6 Y} + \frac{G_r e^{-A_2 Y}}{A_2{}^2 + SA_2 - 0.4 + \omega\tan(\omega\tau)}$$

$\xi_0 = e^{-A_3 Y}, \theta_0 = e^{-A_1 Y}, \xi_1 = e^{-A_4 Y}, \theta_1 = e^{-A_2 Y}$
Where the values of $A_1, A_2, A_3, A_4, A_5, A_6$ are defined in appendix.

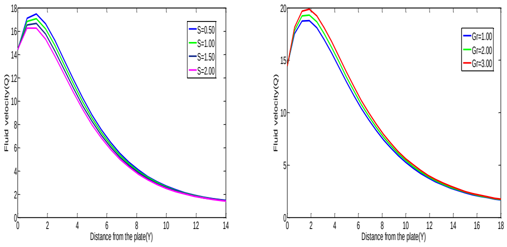
Now putting the values of $Q_0, \xi_0, \theta_0, Q_1, \xi_1, \theta_1$ in the system of coupled ordinary differential equations. We obtain.

$$Q(Y,\tau) = \left(1 - \frac{G_r}{{A_1}^2 + SA_1 - 0.4}\right) e^{-A_5 Y} + \frac{G_r e^{-A_1 Y}}{{A_1}^2 + SA_1 + 0.4} +$$
$$\varepsilon(e^{i\omega\tau} + e^{-i\omega\tau}) \left\{\left(1 - \frac{G_r}{{A_2}^2 + SA_2 - 0.4 + \omega\, tan(\omega\tau)}\right) e^{-A_6 Y}\right\}$$
$$+ \frac{G_r e^{-A_2 Y}}{{A_2}^2 + SA_2 - 0.4 + \omega\, tan(\omega\tau)}$$

$$\xi(Y,\tau) = e^{-A_3 Y} + \varepsilon(e^{i\omega\tau} + e^{-i\omega\tau}) e^{-A_4 Y}$$
$$\theta(Y,\tau) = e^{A_1 Y} + \varepsilon(e^{i\omega\tau} + e^{-i\omega\tau}) e^{A_2 Y}$$

V. RESULT

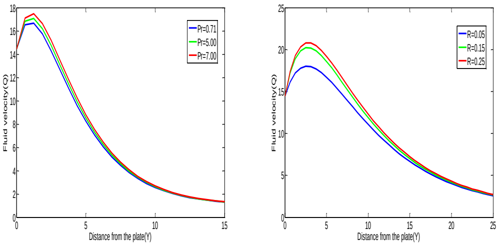
The main goal of the computation is that the analytical solutions of fluid velocity Q, magnetic induction ξ, fluid temperature θ changes with different values of Suction parameter S, Grashoff numberG_r, prandtl number (P_r), Magnetic Diffusivity Number(P_m) , Absorption Parameter(β), Radiation parameter (R).In Figure 5.1(i) shows the velocity profiles for different values of Suction Parameter(S). This figure shows that the velocity profiles decrease with the increase of$S$.In Figure 5.1(ii) shows the velocity profiles for different values of Grashof number(G_r). This figure shows that the velocity profiles increase with the increase ofG_r.In Figure 5.1(iii) shows the velocity profiles for different values of Prandtl number(P_r). This figure shows that the velocity profiles increase with the increase ofP_r.In Figure 5.1(iv) shows the velocity profiles for different values of radiation parameter(R). This figure shows that the velocity profiles increase with the increase of R.



(a) (b)

Figure: 5.1(a) Variation in velocity with Suction Parameter S at $P_r = 0.71, \beta = -0.5, R = 0.05, G_r = 1.00, \omega\tau = \pi/4$

Figure: 5.1(b) Variation in velocity with Grashof Number $G_r$at $P_r = 0.71, \beta = -0.5, R = 0.05, S = 0.50, \omega\tau = \pi/4$

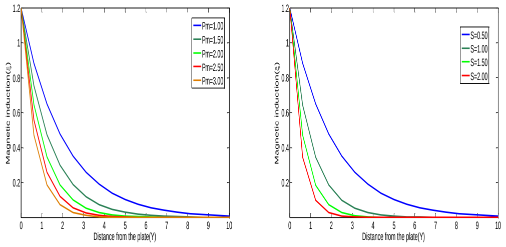


(c) (d)

Figure: 5.1(c) Variation in velocity with Prandtl Number P_r at $G_r = 1.00, \beta = -0.5, R = .05, S = 0.50, \omega\tau = \pi/4$

Figure 5.1(d) Variation in velocity with Radiation Parameter R at $P_r = 0.71, \beta = -0.5, G_r = 1.00, S = 0.50, \omega\tau = \pi/4$

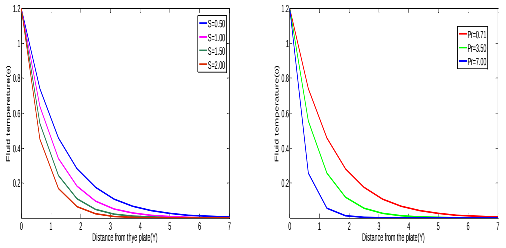
In Figure 5.2(e) shows the magnetic induction profiles for different values of Suction parameter (P_m). This figure shows that the magnetic induction profiles decrease with the increase of P_m. In Figure 5.2(f) shows the magnetic induction profiles for different values of Magnetic diffusivity number (P_m). This figure shows that the magnetic induction profiles decrease with the increase of S.



(e) (f)

Figure: 5.2(e) Variation in magnetic induction with Suction parameter S at $P_m = 1.00, \omega\tau = \pi/4$

Figure 5.2(f) Variation in Magnetic induction with Magnetic diffusivity number P_m at $S = 0.50, \omega\tau = \pi/4$

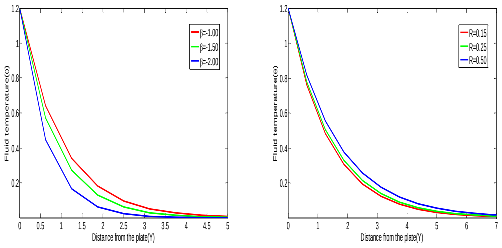


(g) (h)

Figure: 5.3(g) Variation in temperature with Suction Parameter S at $G_r = 1.00,\ P_r = 0.71,$ $\beta = -0.50, R = 0.05, \omega\tau = \pi/4$

Figure: 5.3(h) Variation in temperature with Prandtl Number P_r at $G_r = 1.00, S = 0.50,\ \beta = -0.50, R = 0.05, \omega\tau = \pi/4$

In Figure: 5.3(iii) shows the temperature profiles for different values of Absorption Parameter(β). This figure shows that temperature profiles decrease with the increase ofβ.In Figure: 5.3(iv) shows the temperature profiles for different values of Radiation Parameter(R). This figure shows that temperature profiles increase with the increase of R.



(j) (k)

Figure 5.3(j) Variation in temperature with Heat Generation or Absorption parameter β at $G_r = 1.00$, $S = 0.50$, $P_r = 0.71, R = 0.05$, $\omega\tau = \pi/4$

Figure: 5.3(k) Variation in temperature with Radiation Parameter R at $G_r = 1.00, S = 0.50, P_r = 0.71, \beta = -0.50,$ $\omega\tau = \pi/4$

Appendix

$$A_1 = \frac{1}{2}\left[\frac{SP_r + P_r\sqrt{S^2 - 4(\beta + R\beta)/P_r}}{1+R}\right],$$

$$A_2 = \frac{1}{2}\left[\frac{SP_r + P_r\sqrt{S^2 - 4(1+R)(\beta + \omega\tan(\omega\tau))/P_r}}{1+R}\right],$$

$$A_3 = SP_m,$$

$$A_4 = \frac{1}{2}\left\{SP_m + P_m\sqrt{S^2 - \frac{4\omega\tan(\omega\tau)}{P_m}}\right\},$$

$$A_5 = \frac{1}{2}\left\{S + \sqrt{S^2 + 1.6}\right\},$$

$$A_6 = \frac{1}{2}\left\{S + \sqrt{S^2 - 4(-0.4 + \omega\tan(\omega\tau))}\right\}$$

Condition

$\alpha = 90^0, \lambda = \cos 90^0 = 0$ and $(0, iR') = (0,0.2)$

VI. REFERENCE

[1] Monika M, Net R, Mukesh K (2016) “Radiative Effect on Flow and Heat Transfer over a Vertically Oscillating Porous Flat Plate Embedded in Porous Medium with Oscillating Surface Temperature”, Scientific Research Publishing (Open Journal of Fluid Dynamics) 2016, 6, P- 119-129

[2] Tanvir A and Mahmud A**,** (2013) “Finite Difference Solution of MHD Mixed Convection Flow with Heat Generation and Chemical Reaction**”** Procedia Engineering (Elsevier Journal), **Vol-56**, pp-149-156.

[3] Tanvir A and Mahmud A**,** (2014) “Chemically Reacting Ionized Fluid Flow through a Vertical Plate with Inclined Magnetic Field in Rotating System**”** Procedia Engineering (Elsevier Journal), **Vol-90**, pp-301-307.

[4] Shahins A (2010)“Induced Magnetic field Radiating Fluid Over A Porous Vertical Plate: Analytical Study”Journal Of Naval Architecture and Marine Engineering,DOI:10.3329/jname.v7i2.5662.

[5] G.V.Ramana R, S Mohammed I, and V.S.Bhagavan, (2014) “Similarity Transformation Of Heat And Mass Transfer Effects On Steady MHD Free Convection Dissipative Fluid Flow Past An Inclined Porous Surface With Chemical Reaction” Journal Of Neval Architecture and Marine Engineering, jname.v11i2.18313.

IJEAST

INTERNATIONAL JOURNAL
OF ENGINEERING APPLIED SCIENCE AND TECHNOLOGY

ABOUT IJEAST

International Journal of Engineering Applied Science and Technology (IJEAST) is a peer-reviewed, open access journal that publishes high-quality research papers in the field of Engineering, Applied Science and Technology.

IJEAST aims to provide a platform for researchers, academicians, and professionals to share their innovative ideas, research findings, and practical experiences with the global scientific community.

FOCUS AREAS

- Engineering
- Applied Science
- Technology
- Innovation & Development
- Interdisciplinary Studies

PEER REVIEWED

All submissions are rigorously peer reviewed to ensure quality.

OPEN ACCESS

Free and unrestricted access to research for all.

GLOBAL REACH

Connecting researchers and professionals worldwide.

TIMELY PUBLICATION

We ensure a swift and efficient publication process.

For more information, visit our website
www.ijeast.com

INTERNATIONAL JOURNAL
OF ENGINEERING APPLIED SCIENCE
AND TECHNOLOGY

editor@ijeast.com
www.ijeast.com
India

2455-2143