



# MODIFIED DENOISING ALGORITHM FOR SIGNALS WITH DIFFERENT NOISE DISTRIBUTIONS

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**Abstract---**Wavelet denoising based on wavelet Transform (WT) is simple and has been the dominant technique in the area of non-parametric signal denoising. The more robust threshold technique used in Wavelet denoising is Universal threshold. In general, the selection of the threshold estimator depends on the data. When the signal SNR is low, this threshold estimator is suboptimal in noise removal. We propose denoising approach based on modified universal threshold algorithm applied separately on to noise dominated detailed coefficients and signal dominated coefficients for better denoising performance. We test this with simulated ECG data set added with four types of noise models, White Gaussian, Flicker, Impulse and Raleigh noise distributions. We compare performance of threshold algorithms using Signal Mean Square Error (SMSE). It is found that the modified approach has a better denoising performance.

**Keywords--**Wavelet Transform, Universal Threshold

## I. INTRODUCTION

To solve the problem of Blind Source Separation (BSS) of signals with additive noise, a good solution is to apply a powerful denoising processing before signal separation. At present denoising techniques mainly include Wavelet denoising and Empirical Mode Decomposition (EMD) denoising [1]. Denoising based on Wavelet Transform is simple and dominant in which signal is decomposed using functions (wavelets) well localized in both physical space (time) and spectral space (frequency) generated from each other by translation and dilation,

enabling investigation of periodic and transient signals.

The majority of wavelet algorithms use decimated discrete decomposition of the signal to concentrate information in some wavelet coefficients. The denoising idea is to conserve only the greatest (signal dominated) coefficients and put others (corresponding to noise) at zero before construction of the signal. Thresholding step modifies and process all of the discrete detail coefficients (DC) so as to remove noise. From the beginning of usage of wavelet transforms in signal processing, it has been found that wavelet thresholding is of considerable interest for removing noise from signal.

In this paper, we propose a modified universal threshold algorithm to denoise signals with strong noise. This approach can apply the modified thresholds separately to noise-dominated coefficients and signal-dominated coefficients and then denoise the signal. In order to compare the effect of this method in denoising of noise components (color noise and spatial noise) we have constructed four kinds of stochastic models: 1) The pure white Gaussian noise model 2) Flicker or Pink color noise model 3) Impulse noise model and 4) Rayleigh noise model.

This paper is organized as follows: In section 2, we introduce different types of noises and four noise models.

Then in section 3, we explain the Wavelet Analysis and Threshold algorithms. In section 4 we analyze the disadvantages of conventional Wavelet denoising, briefly describe Hurst coefficient algorithm and find a new (modified) denoising approach which has better performance. Finally, we apply new denoising



approach to noisy ECG signal and summary concludes this paper.

**II. NOISE TYPES**

By definition, Noise is the part of the observation that masks the underlying signal we wish to analyze by lowering its Signal to Noise Ratio (SNR), and in itself adds no information to the analysis. However, for a noise signal to carry no information, it must be WHITE with flat spectrum and an autocorrelation function equal to an impulse. Most real noise is not really white, but colored in some respect. The following Color or spectrum noises and spatial noises and their effect on denoising of signals are investigated.

**a) COLOUR NOISE:**

The color of a noise signal (a signal produced by a stochastic process) is generally understood to be some broad characteristic of its power spectrum. Different colors of noise have significantly different properties: for example, as audio signals they will sound differently to human ears, and as images they will have a visibly different texture. Therefore, each application typically requires noise of a specific color. The practice of naming kinds of noise after colors started with white noise, a signal whose spectrum has equal power within any equal interval of frequencies. That name was given by analogy with white light, which was (incorrectly) assumed to have such a flat power spectrum over the visible range. Other color names, like pink, red, blue and violet were then given to noise with other spectral profiles.

**b) SPATIAL DISTRIBUTED NOISES:**

Often the signals are corrupted by **spatial distributed noises** during transmission due to interference in the channel: Impulse noise and Rayleigh noise [2] or fading is a statistical model for the effect of a propagation environment on a radio signal, such as that used by wireless devices. Rayleigh fading models assume that the magnitude of a signal that has passed through such a transmission medium will vary randomly, or fade, according to an Impulse or Rayleigh distribution — the radial component of the sum of two uncorrelated Gaussian random variables.

**c).FOUR NOISE MODELS**

**Model 1) White Gaussian Noise (WGN) Model:**

**WGN** is basic and general model for thermal noise in communication channels. It is the set of assumptions that the noise is additive, i.e., the received signal equals the transmit signal plus some noise, where the noise is statistically independent of the signal, the noise is white, i.e., the power spectral density is flat, so the autocorrelation of the noise in time domain is zero for any non-zero time offset and the noise samples have a Gaussian distribution.

Pdf of Gaussian distribution is

$$p(z) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(z-\mu)^2/2\sigma^2} \dots\dots\dots (1)$$

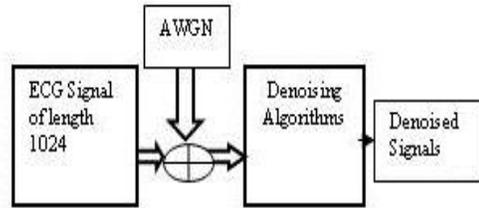


Figure 1 White Gaussian Noise model

In this model, ECG of length 1024 are generated and AWG Noise with mean=0 and  $\sigma=1.0$  is added to the ECG signal. Noise is varied in 2dB steps from -4dB to +16dB. Noisy ECG signal is then denoised using wavelet Transform algorithms and results are compared for performance using SMSE criterion.

**Model 2) Flicker Noise or Pink color noise Model:**

The flicker noise is sometimes used to refer to pink noise, although this is more properly applied only to its occurrence in electronic devices. Pink noise or 1/f noise is a signal or process with a frequency spectrum such that the power spectral density (energy or power per Hz) is inversely proportional to the frequency of the signal. In pink noise, each octave (halving/doubling in frequency) carries an equal amount of noise power. The name arises from the pink appearance of visible light with this power spectrum.

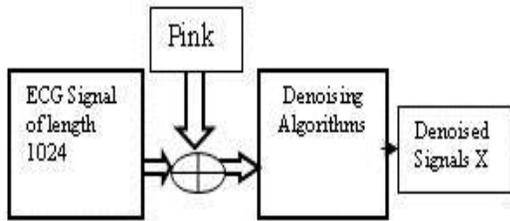


Figure 2 Ficker or pink noise model

In this model, ECG of length 1024 are generated and Pink color Noise '1/f' is added to the ECG signal. Noise is varied in 2dB steps from -4 dB to +16dB. Noisy ECG signal is then denoised using wavelet Transform algorithms and results are compared for performance using SMSE criterion.

**Model 3) Impulse Noise Model:**

Impulse noise or spike noise also called salt-and pepper noise is usually caused by timing errors in the process of digitization, faulty memory locations, malfunctioning of pixel elements in Camera sensors. Impulse noise of equal height impulses is called salt-and pepper noise and there are only two possible values exist that is a and b the probability of each is less than 0.2. Impulse noise of un equal height impulses is called random values impulse noise is used in this model. The probability density function of impulse noise is

$$p(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases} \dots\dots\dots$$

(2)

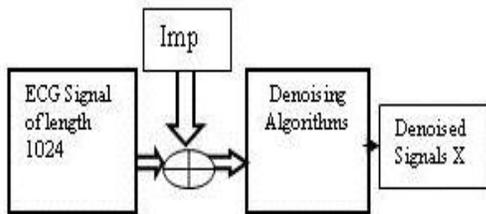


Figure 3 Impulse noise model

In this model, ECG signal of length 1024 are generated and Impulse Noise with mean=0 and

$\sigma=1.0$  is added to the ECG signal. Noise is varied in 2dB steps from -4dB to +16dB .Noisy ECG signal is then denoised using wavelet Transform algorithms and results are compared for performance using SMSE criterion.

**Model 4) Rayleigh Noise Model:**

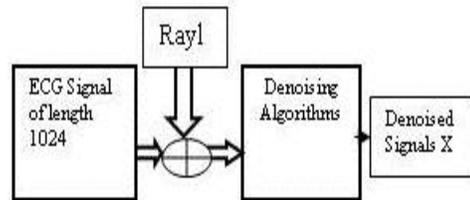


Figure 4 Rayleigh noise model

Rayleigh fading is viewed as a reasonable model for tropospheric and ionospheric signal propagation as well as the effect of heavily built-up urban environments on radio signals. Rayleigh fading is most applicable when there is no dominant propagation along a line of sight between the transmitter and receiver. How rapidly the channel fades will be affected by how fast the receiver and/or transmitter are moving. The probability density function of the Rayleigh distribution is

$$p(z) = \begin{cases} \frac{2}{b(z-a)e^{-\frac{(z-a)^2}{b}}} & \text{for } z \geq a \\ 0 & \text{for } z < a \end{cases}$$

$$\sigma^2 = b(4 - \pi)/4, \quad \mu = a + \sqrt{\pi b}/4 \dots\dots\dots$$

(3)

In this model, in this model, ECG of length 1024 are generated and Rayleigh Noise with mean=0 and  $\sigma=1.0$  is added to the ECG signal. Noise is varied in 2dB steps from -4 dB to +16dB .Noisy ECG signal is then denoised using wavelet Transform algorithms and results are compared for performance using SMSE criterion.



### III. WAVELET ANALYSES

The wavelet transform is very useful tool in the analysis of non stationary signals such as the reflection seismic signals. The theory and methods of wavelet analysis are detailed in books [3],[4].In this paper, discrete wavelet analysis is used instead of the continuous wavelet analysis. The discrete wavelet analysis is based on the concept of Multi-resolution analysis (MRA) introduced by Mallat [5].With the MRA, a signal is decomposed recursively into sum of details and approximations at different levels of resolution as shown in Figure 5

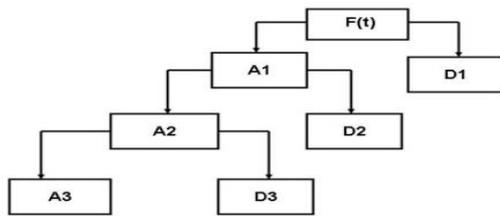


Figure. 5. Decomposition tree of signal  $f(t)$ ,  $D_i$  and  $A_i$  are details and approximation components at level  $i$ .

The details represent the high frequency components while the approximations represent the low frequency components of the signal. The decomposition algorithm is fully recursive. At each stage of MRA the signal is passed through a High pass filter called scaling filter, denoted as  $G$  and a Low pass filter called the wavelet filter, denoted as  $H$ .

These filters are quadrature mirror filters that satisfy the orthogonality conditions;  $HG^*=GH^*=0$  and  $H^*H+G^*G=I$ ; where  $I$  is the identity operator. The filters  $H$  and  $G$  are the decomposition filters, while the filters  $H^*$  and  $G^*$  are the reconstruction filters.

The coefficients of the filters  $H$  and  $G$  depend on the particular wavelet used for the decomposition [6].The process of decomposing a signal  $f(t)$  and reconstructing the approximations  $A_i$  and  $D_i$  is shown in Figure 6.

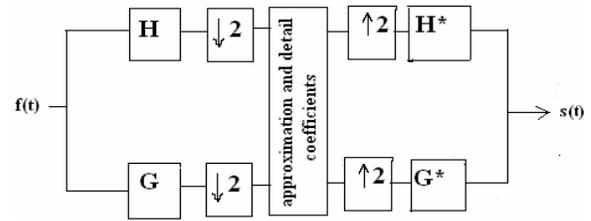


Figure. 6. The process of decomposition and reconstruction of approximations ( $A_i$ ) and details ( $D_i$ ) (level  $i$ ). Symbols  $\downarrow 2$   $\uparrow 2$  represent dyadic down-sampling and up-sampling

As shown in figure 6, the discrete wavelet transform (DWT) analyzes the signal at different frequency bands and with different resolutions by decomposing the signal in to coarse approximation and detail information. The approximation components are obtained by passing the signal through the low pass filter  $H$ , which removes the high frequency components. At this stage, the resolution is halved but the scale remains unchanged. Then, the signal is sub-sampled, thereby removing half the redundant samples. It should be noted that this process does not affect the resolution but affects the scale, which is doubled. Similarly the detailed coefficients are obtained by passing the signal through the high pass filter  $G$ . his constitutes one level of de-composition. The wavelet coefficients thus obtained can then be used for the purposes of signal de-noising and compression [7].

#### A. Threshold Algorithms

To denoise a signal using wavelet transforms the detail coefficients are thresholded [8]. The simplest threshold technique is the hard threshold, where the new values of the details coefficients  $d(t)$  are found according to the following:

$$d(t) = \begin{cases} d(t) & \text{if } |d(t)| > \theta \\ 0 & \text{if } |d(t)| \leq \theta \end{cases} \quad \text{----- (4)}$$

Where  $d(t)$  are detailed coefficients and  $\theta$  is the threshold

Another method of threshold is the soft threshold, where the new details coefficients are given by the following:



$$d^{\xi}(t) = \begin{cases} \text{sign}(d(t))(|d(t)| - \theta) & \text{if } |d(t)| > \theta \\ 0 & \text{if } |d(t)| \leq \theta \end{cases} \quad \text{-- (5)}$$

The threshold  $\theta$  can be estimated using different threshold estimating algorithms and widely used Universal Threshold is as follows:

$$\theta = \sigma \sqrt{2 \log(N)} \quad \text{----- (6)}$$

Where  $N$  is the length of threshold coefficients and  $\sigma$  characterizes the noise level.

Hard threshold can be described as the usual process of setting to zero the elements whose absolute values are lower than the threshold. Soft threshold is an extension of hard threshold, first setting to zero the elements whose absolute values are lower than the threshold, and then shrinking the nonzero coefficients towards 0. Threshold selection rules and algorithms are described in next section.

**I. Universal Threshold**

This type of global thresholding method was proposed by Donoho and Johnstone. This is also called Universal threshold method. The threshold value is given in equation (6), where  $N$  is the number of data points, and ‘ $\sigma$ ’ is an estimate of the noise level. Donoho and Johnstone proposed an estimate of  $\sigma$  that is based only on the empirical wavelet coefficients at the highest resolution level ( $j - 1$ ) because they consist most of noise. Most of the function information except the finest details is in lower level coefficients. The median of absolute deviation (MAD) estimator is expressed in equation (6) as

$$\text{median}(|W_{j-1,k} - \text{median}(W_{j-1,k})|) / 0.6475 \quad \dots (7)$$

The universal thresholding removes the noise efficiently. The fitted regression curve is often very smooth and hence visually appealing. If  $z_1 \dots z_n$  represent the wavelet coefficients of the noise with  $\text{iid } N(0, \sigma^2)$ , then it is expressed in equation (7) as

$$\lim_{n \rightarrow \infty} P(z_i \leq \hat{\sigma} \sqrt{2 \log n}) = 1 \quad \dots (8)$$

This means that the probability of all noise being shrunk to zero is very high for large samples. Since the universal thresholding procedure is based on this

asymptotic result, it sometimes does not perform well in small sample situations.

**B DENOISED SIGNAL RECONSTRUCTION**

Because of the amplitude based muting (based on thresholds) wavelet-transform based filters are in general nonlinear and can be readily applied to non-stationary reflection signals. Wavelet filters are efficient for filtering several types of noise in seismic data at the same time. Reconstruction or synthesis is the process of assembling those components back into the signal. The mathematical manipulation that affects synthesis is called: the *inverse discrete wavelet transforms* (IDWT). In order to get the de-noised signal, the new details coefficients,  $d^{\xi}(t)$ , are used in signal construction process instead of original coefficients  $d(t)$ . The de-noised procedure is summarized in Figure 7.

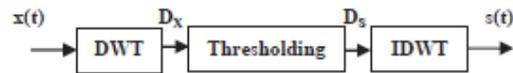


Figure 7. DWT de-noising procedure

**IV. MODIFIED DENOISING APPROACH**

**A. Disadvantages of Wavelet Denoising**

The general wavelet de-nosing procedure starts with first selection of a wavelet that may not match varying nature of signals resulting in drawback. However, to some extent this drawback can be minimized based on eyeball inspection of the signal with noise, or it can be selected based on correlation  $\gamma$  between the signals of interest.

The second disadvantage wavelet denoising is decomposition of signal in to coefficients based on fixed level irrespective of Signal SNR is high or low. Thus the decomposition level is non adaptive.

The third disadvantage wavelet denoising is application of threshold levels universal to all levels disregarding noise dominated or signal dominated coefficients. Hence denoising approach cannot remove the noise efficiently.



**B. The Hurst Coefficient**

In this section, we propose a new denoising approach. This approach is based on Hurst coefficient proposed by H. E. Hurst [9] for denoising use. The Hurst exponent provides a measure for long term memory and fractality of a time series. Since it is robust with few assumptions about underlying system, it has broad applicability for time series analysis. The values of the Hurst exponent range between 0 and 1. Based on the Hurst exponent value H, a time series can be classified into three categories. (1)  $H=0.5$  indicates a random series. (2)  $0 < H < 0.5$  indicates an anti-persistent series or noise dominated (3)  $0.5 < H < 1$  indicates a persistent series. An anti persistent series has a characteristic of “mean-reverting”, which means an up value is more likely followed by a down value, and vice versa. The strength of “mean reverting” increases as H approaches 0.0. A persistent series is trend reinforcing, which means the direction (up or down compared to the last value) of the next value is more likely the same as current value. The strength of trend increases as H approaches 1.0.

The best-known method to estimate the Hurst exponent is R/S analysis. It was proposed by Mandelbrot and Wallis [10], based on the previous work of Hurst.

The procedure is as follows.

The time series of length L has to be divided into d sub series ( $Z_{i,m}$ ) of length n, and for each sub series  $m = 1, \dots, d$ . Then,

1. It is necessary to find the mean ( $E_m$ ) and the standard deviation ( $S_m$ ) of the sub series ( $Z_{i,m}$ ).
2. The data of the sub series ( $Z_{i,m}$ ) has to be normalized by subtracting the sample mean  $X_{i,m} = Z_{i,m} - E_m$  for  $i=1, \dots, n$
3. Create the cumulative time series  $Y_{i,m} = \sum_{j=1}^i X_{j,m}$  for  $i = 1, \dots, n$ .
4. Find range  $R_m = \max\{Y_{1,m}, \dots, Y_{n,m}\} - \min\{Y_{1,m}, \dots, Y_{n,m}\}$
5. Rescale the range ( $R_m/S_m$ ).
6. Calculate the mean value  $(R/S)_n$  of the rescaled range for all sub series of length n.

Considering that the R/S statistic asymptotically follows the relation  $(R/S)_n \approx c n^H$ , the value of H can be obtained by running a simple linear regression over a sample increasing time horizons.

$$\log (R/S)_n = \log c + H \log n \dots\dots\dots (9)$$

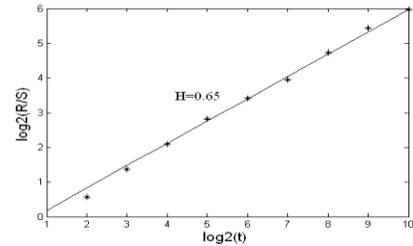


Figure 8 An example of R/S analysis

**C. Modified Threshold Method**

In this section, we propose a new modified universal threshold algorithm to denoise signals. The noisy signal is first decomposed with the detail coefficients and the approximation coefficients. Low-frequency components are shown with large coefficients and high frequency components are shown with small coefficients. Wavelet coefficients that is smaller than the threshold value is removed. As a result, the original signal is obtained from noisy signal.

However, we find the thresholds decrease so slowly from the first to the last part of the signal may get lost from some signal-dominated detail coefficients after thresholding. Therefore, we should first find the detail coefficients which are dominated by the noise and the ones dominated by the signal. Then we apply the universal threshold to the noise dominated detail coefficients, and another estimating threshold method which is able to decrease faster to the signal dominated detail coefficients..

The proposed modified denoising approach consists of three key steps: firstly, find the detail coefficients dominated by noise and threshold them separately; secondly, compute the thresholds of other detail coefficients using a new estimating threshold which is able to decrease faster; lastly, apply the thresholding technique to each detail coefficient, and then reconstruct the signal by adding the thresholded detail coefficients. Detail explanations are:

- 1) Find the detail coefficients dominated by the noise.



We have introduced Hurst exponent method to find the noise dominated detail coefficients. Based on the Hurst exponent value H, a time series can be classified into an anti-persistent series if its H value is between 0 and less than 0.5. Therefore, in practice, in deciding whether the detail coefficient is noise dominated or not is judged by its Hurst coefficient value H. Then we use the Universal threshold method to estimate threshold level and threshold each noise dominated detail coefficients.

2) Get the thresholds of the other Detail Coefficients.

We have to find another estimation method to get the thresholds of the signal dominated detail coefficients. Since the estimation of Universal threshold decrease slowly or more or less remain constant, we need one which can decrease faster for signal dominated detail coefficients.

First, through studying the Universal threshold

$$\theta = \sigma \sqrt{2 \log(N)} \dots\dots\dots (10)$$

$$\text{Thr}_{i=\theta} = C \sqrt{(V/\beta)} \cdot \rho^{-i} \cdot 2 \log N \dots\dots\dots (11)$$

V is the energy and  $\beta$ ,  $\rho$  are the parameters [11] and equal to 0.179 and 2.01 respectively when the noise is white Gaussian noise.

Removing constant term and substituting  $\rho \approx 2$

$$\text{Thr}_{i(\text{new})} \propto (\sqrt{2})^{-i} \dots\dots\dots (12)$$

In order to make this threshold decreased faster, the form of new modified threshold that we are proposing will be an exponential function. and we set

$$\text{Thr}_{i(\text{new})} \propto \square^{-i}, \quad i=1,2,3,4,\dots\dots\dots (13)$$

The new threshold function needs to be proportional to the threshold of the last noise dominated detail coefficient 'k'. Thus the expression of the threshold is set as:

$$\text{Thr}_{i(\text{new})} = \text{Thr}_k / \square^{i-k}, \quad i=k+1,\dots\dots\dots K \dots\dots\dots (14)$$

In order to make the threshold decrease faster  $\square > \sqrt{2}$  has to be satisfied. For this,  $\square$  is selected as 1.6, 1.8, 2, 2.2, and 2.4 for noisy signal denoising. The

value of  $\square$  is determined by the denoising experiment and the best results are obtained when  $\square=2$ , So, the threshold is expressed as follows.

$$\text{Thr}_{i(\text{new})} = \text{Thr}_k / 2^{i-k}, \quad i = k+1 \dots K. \dots\dots (15)$$

For example,  $K=12$  and  $k=1$ , comparing the conventional Universal threshold with the new modified threshold (Figure 9), we can see that the new thresholds decrease faster than Universal threshold which is nearly constant.

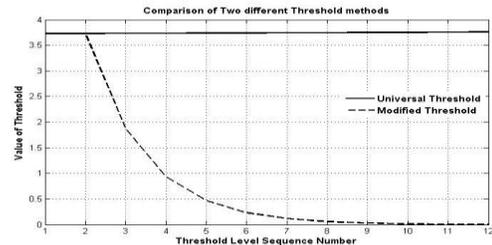


Figure 9 Comparison of Two threshold methods

3) Use the estimated thresholds to each detail coefficient

Now the threshold of each detail coefficient is being determined, threshold the signal dominated detail coefficients and then reconstructs the signal.

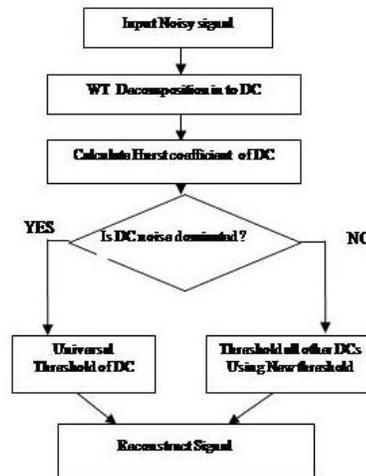


Figure. 10. New WT modified threshold denoising approach Scheme



**V. EXPERIMENTAL RESULTS**

The experimental analysis of this section aims at objectively evaluating the de-noising performance of Universal threshold estimating algorithm namely sqtwolog, and the performance of New threshold algorithm. In order to precisely describe the performance of the denoise algorithms, we employ **signal mean square error (SMSE)**, a contrast-independent criterion defined as

$$SMSE = \frac{1}{N} \sum_{j=1}^N E \{ |x_j - \hat{x}_j|^2 \} \dots\dots\dots (16)$$

Where  $x_j$  is the source signal or the noise free signal,  $\hat{x}_j$  is estimated signal, and N is sample number of the signal. The performance is better when the value of SMSE is smaller.

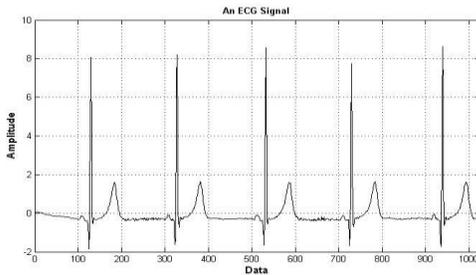


Figure 11 an ECG Signal

We performed numerical simulation of ECG test signal using MATLAB as shown in figure 11. The sample size of the signals is N=1024. The parameters of WT are set as the basis function is “sym7” and the number of decomposition levels is 12. In order to compare the effect of Universal threshold algorithm with new modified threshold approach in denoising of noise components we have constructed Four kinds of stochastic models 1) White Gaussian Noise model 2) Flicker noise model 3) Impulse noise model and 4) Rayleigh noise model.

Figure 12 shows the SMSE comparison for White Gaussian Noise model. The new modified approach is very efficient and outperforms universal threshold algorithm. Comparison of two types of threshold algorithms shows that both threshold algorithms could reduce the distribution of white Gaussian noise up to SNR of 16 dB with modified threshold algorithm performing better. For SNR from -2dB to -4dB the performance of each algorithm differed with steep rise in their SMSE. Modified threshold

algorithm performing better in the cases where the signal SNR is low.

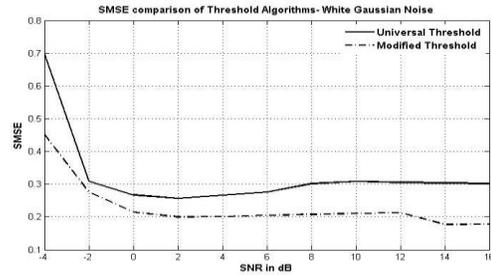


Figure 12 Denoising performance of threshold algorithms for White gaussian noise

Figures 13,14 and 15 show the comparison of threshold algorithms for the noise model 2,3 and 4 respectively. It can be seen by comparison of figures 13, 14, and 15 that there is significant effect of threshold algorithms in denoising of noise models of Flicker noise, Impulse noise and Raleigh noise.

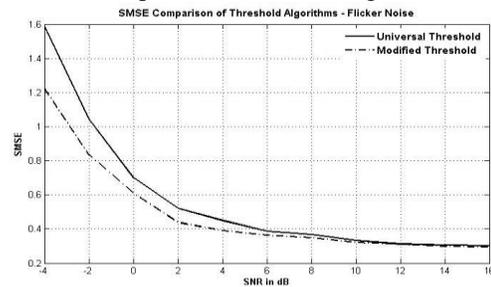


Figure 13 Denoising performance of threshold algorithms for flicker noise

In model 2 we can observe the denoising performance is poor compared to model 1 with modified threshold approach performing better. The performance remained same up to 8dB SNR and from 8dB SNR to -4dB SNR modified threshold performed better.

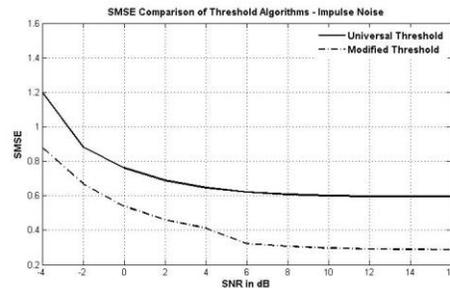


Figure 14 Denoising performance of threshold algorithms for impulse noise



In noise models 3 and 4, we can observe the performance of Universal Threshold algorithm differ from that of modified threshold algorithm throughout the SNR range starting from 16dB to -4dB. In model 3 the worst case SMSE is 0.8 for modified threshold compared to 1.2 of Universal Threshold algorithm.

In noise model 4, Modified threshold performed better throughout the SNR range starting from 16dB to -4dB. The performance of modified threshold algorithm is better for high SNR compared to low SNR.

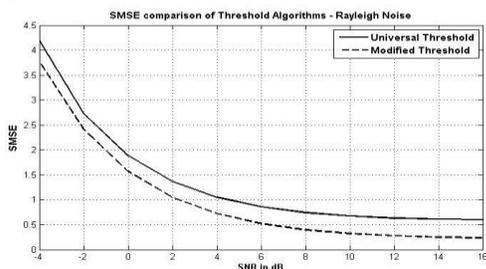


Figure 15 Denoising performance of threshold algorithms for Rayleigh noise

## VI. CONCLUSIONS

In this paper we applied two thresholding algorithms, Universal Threshold and modified Threshold algorithm in an aim to impact of thresholding in denoising of ECG signals. In order to compare the effect of them in denoising of noise components we have constructed four kinds of stochastic models: the white gaussian noise model (1), the Flicker noise model(2),the Impulse noise model (3) and Rayleigh noise model (4). SMSE criterion is used for performance measurement. The results are:

1. In model (1) and (2), both threshold algorithms could reduce the distribution of white gaussian noise and Flicker noise in different degree. New modified threshold algorithm could remove almost complete noise for SNR from 16dB to -4dB while algorithms could remove noise reasonably for SNR from 16dB to 0dB then the denoising performance exponentially degraded for SNR from 0 dB to -4 dB.

2. In model (3) and (4), from Figures 14 and 15, both threshold algorithms could reduce the distribution of Impulse and Rayleigh noise in more or less same

level with new modified threshold algorithm performing better. Both algorithms could remove complete noise for SNR starting from 16dB to 0 dB and their performance exponentially degraded for SNR between 0 dB and -4dB.

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