

SLOPE STABILITY ANALYSIS TECHNIQUES: A REVIEW

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Abstract— Analysis of slopes for stability and safety is a major area of concern in civil engineering. This is the reason that so many analysis techniques have been developed so far. Traditional way of slope stability analysis involve the determination of factor of safety for a slope to take safety precautions against any instability. Various researchers worked to develop a new method in which probability of failure or reliability of a slope is calculated.

Thus, basically two different approaches of slope stability analysis are available with us— deterministic approach and probabilistic (or reliability) approach. In this paper past trends in slope stability analysis are discussed with the evolution of each method. A brief review of available methods has also been presented here along with the advantages and limitations of their use.

Keywords— Slope Stability, Deterministic, Reliability, Optimization.

I. INTRODUCTION

Stability of natural slopes and man-made slopes such as roads/railways embankment, hydraulically constructed dams, earth dams etc. is a major issue in geotechnical engineering. Traditional method used for slope stability analysis is 'limit equilibrium method', in which a single value of factor of safety is calculated to predict the stability of slope. Afterwards, some researchers developed finite element methods as a powerful technique in analyzing the slope stability problems. But the problem of slope stability is related to risk and reliability. Thus a single factor of safety cannot be relied on for taking safety measures against failure. Reliability analysis of slopes involve the calculation of reliability Index for a slope or alternatively probability of failure of a slope.

In both the above given approaches the very important part is the search of critical slip surface i.e. critical deterministic slip surface or critical probabilistic slip surface which is a constraint optimization problem. Various optimization techniques have their advantages in solving slope stability problems. Ranging from simple optimization techniques, such as linear, non-linear programming, quadratic programming, dynamic programming, interior point method etc., the advanced techniques such as simulated annealing, artificial

intelligence algorithms have been successfully used for slope stability analysis. With the advancement of computers it become easy to implement any of these methods.

II. DETERMINISTIC METHODS

Deterministic approach involve various techniques such as: limit equilibrium methods (LEMs), limit analysis (LA), finite element analysis (FEM) and finite difference method (FDM). In limit equilibrium methods the equilibrium of a soil mass tending to slip under the influence of gravity is investigated. Failure in this method is described as the condition when driving forces (or moments) exceeds the resisting forces (or moments).

Moment equilibrium is generally used for the analysis of rotational landslides. The factor of safety with respect to moment is defined as 'F_m' and given by:

$$F_m = \frac{M_r}{M_d} \quad (1)$$

Where,

M_r = the sum of the resisting moments and
M_d = the sum of the driving moment.

In case of circular failure surface, the moment point for convenience is taken as the center of the circle (of which slip surface is a part) and for non-circular failure surface, an arbitrary point may be taken in the analysis.

Force equilibrium is generally applied to translational or rotational failures. In such cases failure surface is either planar or polygonal.

The corresponding factor of safety 'F_f' defined with respect to force is given by:

$$F_f = \frac{\zeta_f}{\zeta_m} \quad (2)$$

Where,

ζ_f = the available shear strength of soil and
ζ_m = the shear stress needed to mobilize the slip.

Simplified methods are not able to satisfy both the equilibriums simultaneously, as this approach is statically indeterminate. Thus the assumptions are made when equations

for the potential collapsing bodies are assembled. Generally for limit equilibrium analysis, the sliding body is divided into 'n' smaller vertical slices and the method is named as method of slices. Then the tangential and normal stress at the bottom of each section of sliding surface is determined by the analyzing the equilibrium conditions of forces acting on each of the section.

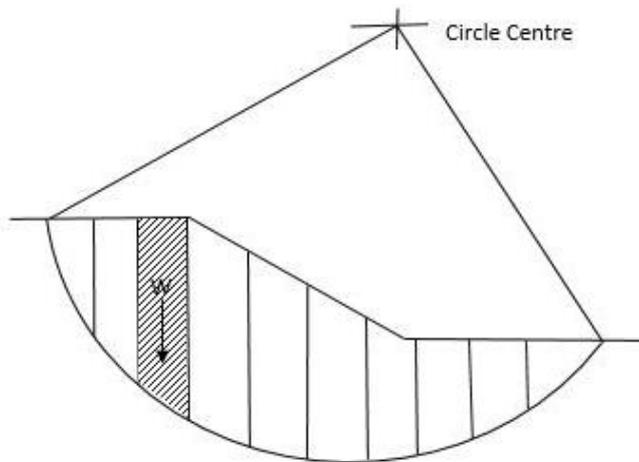


Fig. 1. Method of Slices

Various researchers used different assumptions to make the system of forces determinate and thus obtain different equilibrium equations. Fellenius[1] give the simplest solution of slope stability problem by ignoring all the interslice forces. This method does not satisfy the equilibrium of individual slices and thus leads to inconsistent calculation of effective stresses at the base of slices. After this Bishop[2] developed the equations which satisfy the vertical force equilibrium and overall moment equilibrium about the center of the circular trial surface. But the major limitation of this method is that it cannot be used for noncircular surfaces.

Janbu[4] method satisfy the vertical force equilibrium for each slice as well as overall horizontal force equilibrium for the entire slide mass but do not satisfy the moment equilibrium of slide mass. The system considered by Janbu is over-determined, thus a correction factor f_0 is considered to account for this inadequacy. Morgenstern-Price[5] method satisfy all the three equilibrium conditions i.e. horizontal vertical and moment equilibrium. The interslice force is considered to be inclined and its direction is defined using an arbitrary function. This introduces an additional unknown in the system of equilibrium equations. Spencer[6] gives the similar method as above which can also be used for arbitrary shape of failure surface. This method considered a constant but unknown inclination of resultant of interslice force.

Sarma[7] apply the shear strength criterion to the shears on the sides and bottom of each slice. And varies the inclinations of

the slice interfaces until a critical criterion is met. This method also satisfy all the three equilibrium conditions.

Simplicity of method of slices is its greatest advantage. For simple cases it gives good results and also it is economical to use. But for complex geometries calculation work becomes difficult as it involves trial and error to satisfy the equilibrium equations. Also a number of assumptions are required to make a system determinate in limit equilibrium methods.

To avoid these limitations some researchers introduced the finite element (FE) method with Elasto-plastic soil models for slope stability. One of the earliest studies that used FEM for stability analysis of slopes involved assumption of $\Phi_u = 0$, Smith and Hobbs[8]. Analysis of a number of slopes was carried out and a reasonable agreement with Taylor's charts was obtained.

Zienkiewicz et al[9] considered a c' - ϕ' soil slope and obtained good agreement with slip circle solutions. Griffiths[10] used the FE method to show reliable slope stability results for a vast range of soil types and geometric configurations as compared with the charts of Bishop and Morgenstern. Ugai and Leshchinsky[11] yield similar results as with the rigorous limit equilibrium approach for homogenous slopes. Griffiths and Lane[12] used the finite element method in conjunction with an elastic-perfectly plastic (Mohr-Coulomb) stress-strain method. Failure was considered as the situation when no convergence occurs within the specified number of iterations.

For finite element analysis slope is divided in small fragments called elements and a stress-strain relationship is defined for the case. Four types of relationships are generally used i.e. linear elastic, multi-linear elastic, hyperbolic and elasto-plastic. Each relationship has its own advantages and limitations. For example linear elastic stress-strain relationships are simple but they are useful in modelling the behavior of real soils at low stress levels and small strains. Similarly elasto-plastic and elasto-visco-plastic stress-strain relationships model the behavior of soils close to failure, at failure, and after failure more realistically but these are more complex.

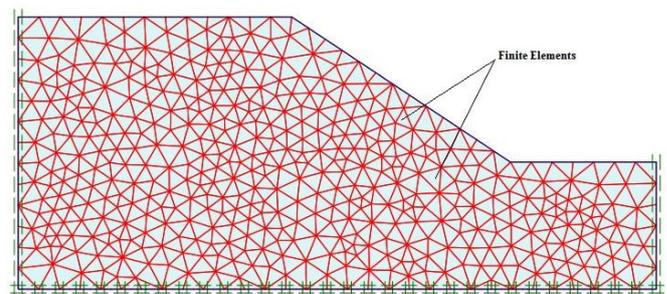


Fig. 2. Finite Element model of a slope



The advantage of using FEM instead of LEMs also includes that the factor of safety emerges naturally in finite element analysis; assumption of particular form of failure mechanisms is not required as in the case of LEMs.

FEM has been used along with more advanced techniques such as strength reduction and gravity increase methods.

Matusai and San[14] used strength reduction method (SRM) for finite element slope stability analysis. It was found that when total shear strain is used, the strength reduction ratio agrees with the factor of safety obtained using the Bishop's method for embankment slopes.

In SRM the strength parameters of the slope are decreased until slope becomes unstable and equilibrium solutions no longer exist. A series of trial factors of safety are used to adjust the strength parameters of soil i.e. cohesion, C and friction angle, ϕ as follows:

$$\begin{aligned} C_{trial} &= \frac{1}{F} C \\ \phi_{trial} &= \arctan\left(\frac{1}{F} \tan \phi\right) \end{aligned} \quad (3)$$

Then the adjusted shear strength parameters of the soil layers (C, ϕ) are re-inputted in the model for equilibrium analysis and factor of safety is calculated when the adjusted value of cohesion and friction angle make the slope unstable, Sternik[15]. SRM generally gives results very similar to LEMs, for the case of homogenous slopes. But Cheng et al[16] found it to be incapable in the determination of other failure surfaces which may be only slightly less critical than the SRM solution. Non-linear shear strength reduction has also been implemented by Fu and Liao[17] in Hoek-Brown shear strength relationship.

Gravity increase method is also a similar technique in which the external forces increases due to increasing gravity, g and the equilibrium solution can no longer be obtained. Sternik[15] explained that the gravity increases according to the formula:

$$g = g' \cdot t \quad (4)$$

Where,

t = parametric time variable.

g' = a prescribed vector specifying the direction of gravity loading and its rate of increase with time.

Thus the procedure involved here is to find the limiting acceleration due to gravity which is given as:

$$g_{limit} = g' \cdot t_{limit} \quad (5)$$

Where,

t_{limit} = largest value of time t , for which solution of global force equilibrium equation for that system exists.

Then the safety factor, F_s is defined as the ratio between the element gravitation in the failure state i.e. limiting acceleration, g_{limit} and the initial element gravitation, g_{actual} , i.e.

$$F_s = \frac{g_{limit}}{g_{actual}} \quad (6)$$

Where, g_{actual} is having the value of 9.81m/s^2

This method was successfully implemented by Li et al[18] in Realistic Failure Process Analysis (RFPA) code using finite element programming.

Finite Elements and finite difference methods were found to be better than limit equilibrium approach but due to the simplicity of formulation of limit equilibrium problem it is still in use. Another method of analysis which is more robust than limit equilibrium and simpler than FE and FD methods is developed known as limit analysis. Donald and Chen[19] uses optimization techniques in a method based on the upper bound theorem of classical plasticity for slope stability analysis. But this method lead to overestimating the factor of safety if the optimization routines fail to find the real or global minimum. Yu et al[20] uses limit analysis approach and modeled the soil as a perfectly plastic material and also obey an associated flow rule using two different theorems to provide a solution: upper bound or lower bound plasticity. Chen et al[21,22] extended the upper bound method developed by Donald and Chen for three dimensional slope stability analysis.

It has been seen in the literature that for the simple cases limit equilibrium methods perform better than finite element method and generally gives lesser factor of safety as compared to FEM. The reason can be the assumptions made during LEM. As in LEM a critical slip surface is assumed and equilibrium equations are made for that particular case but that may not be the critical sliding surface every time. Thus the results by LEM need to be optimized for minimum value of factor of safety, as in simple analyses, calculations are made for prescribed slip surface. This lead to the requirement of more advancement in the analysis methods.

Recent works in deterministic approach includes, incorporating the advanced optimization techniques in existing limit equilibrium and numerical methods.

Bolton et al[23] used leap frog optimization procedure to search for location of critical slip surface in falling mass in Janbu simplified and Spencer methods. This global optimization technique seems to be very useful in locating most critical slope for multi-slope geometry and to find failure surfaces contained within weak layers within the slopes. Chen et al[24] formulated upper bound limit analysis as a non-linear programming problem based on rigid finite elements and used sequential quadratic algorithm to minimize factor of safety.



Method has advantage in modelling non-homogenous soil conditions and complicated boundaries, otherwise gives similar results as of other existing methods.

It has also been observed that heuristic algorithms perform better than simple optimization techniques in stability analysis of slopes having complex geometries. For non-circular slip surfaces simplex or gradient methods can be trapped by a local minima. Cheng et al[25] demonstrated this with the help of a global search technique named particles swarm optimization. Some other powerful optimization techniques had also been applied in slope stability analysis such as genetic algorithm by Zolfaghari et al[26], ant colony optimization, tabu search, simulated annealing, simple & modified harmony search by Cheng et al[27], gravitational search algorithm by Khajehzadeh et al[28] imperialistic competitive algorithm by Kashani et al[29]. A comparison between all mentioned methods has been carried out by Kashani et al[29]. Results showed that imperialistic competitive algorithm gives the least value of FOS for the same problem.

This kind of algorithms enhanced the accuracy in searching the location of critical slip surface and thuds the factor of safety value. But this kind of methods have the limitation of lack of termination criteria. Thus this area still require more intensive research.

From above discussion it can be concluded that the reliable methods are available for the search for critical slip surface and calculating factor of safety for a slope. This calculated factor of safety is then used to design safety measures for that slope. But practically a single factor of safety for whole slope can never exist. Site conditions may vary at different locations. Also various uncertainties may be there in the analysis like uncertainty in soil parameters (c , ϕ , γ), groundwater conditions in different seasons, vegetation and surroundings of the slope etc. might be there. But deterministic approach cannot take into account these all uncertainties in the factor of safety calculations. Thus a more rigorous approach is required to analyze the slopes for stability.

III. RELIABILITY APPROACH

Soil stability can also be defined in terms of risk/probability of failure of slope or reliability of a slope. To account for various uncertainties involved in the analysis of slopes, concept of probability is very reliable to use. The method of analysis is based on the calculation of probability of failure, ' P_f ' or reliability index, ' β ', which are the functions of factor of safety again.

Reliability analysis of slopes and embankments is gaining popularity now-a-days. In past four decades some remarkable work in this field using first-order, second-moment (FOSM) methods include Wu and Kraft[30], Cornell[31], Alonso[32], Tang et al[33], Vanmarcke[34,35], Li and Lumb[36], Luckman et al[37], Halim and Tang[38]. Other than FOSM method the Monte Carlo method is used by Tobutt[39] as a

sensitivity-testing tool for slope stability analysis and also as a method for calculating the probability of failure of a given earth slope. Further Christian et al[40] used mean first order method to explore the use of reliability approach for slope stability analysis and application of probability concepts to account for uncertainties in slope stability parameters.

In these studies reliability index is defined as:

$$\beta = \frac{E(F)-1.0}{\sigma(F)} \quad (7)$$

Where $E(F)$ and $\sigma(F)$ are the statistical parameters of factor of safety i.e. mean and standard deviation, respectively.

Probability of failure is usually calculated on critical deterministic surface by initial researchers. Then Hassan and Wolff[41] found that this critical surface having the minimum factor of safety may or may not be the surface of the maximum probability of failure. Chowdhury and Xu[42], Liang et al[43] and Bhattacharya et al[44] consider the surface with minimum reliability index, β to be the critical slip surface. To locate the critical probabilistic surface optimization of reliability index β , associated with a set of geotechnical parameters including the statistical properties can be done.

Malkawi et al[45] compared first order second moment method (FOSM) and Monte carlo simulation method (MCSM) for calculating reliability index based on various approaches like ordinary method of slices, Bishop method, Janbu method. Results showed that FOSM method requires lesser calculations and computing time but MCSM is more powerful and effective scheme for more detailed reliability analysis of slope stability. Ramly et al[46] also used Monte Carlo simulations for probabilistic analysis of a slope by taking spatial variability of soil parameters into consideration and compared the results with FOSM method and a simplified approach in which spatial variability of soil parameters is ignored. Results showed that method used gives reliability value less than FOSM but more than simplified approach for the case studied whereas simplified approach significantly overestimate the probability of unsatisfactory performance for the slopes that are dominated by uncertainties due to the spatial variability of soil properties.

Griffiths[47] also discovered the similar results by comparing simplified probabilistic approach for finite element analysis and Monte Carlo simulations for finite element nonlinear elasto-plastic analyses i.e. Random finite element method, especially for the cases of low factor of safety and high coefficient of variation of shear strength of soil.

It has also been seen that unless the probability of failure is relatively great, one should not use the advanced first-order second-moment reliability method for evaluating the probability of failure of earth slopes as the reliability of a



slope can be sensitive to the adopted probability distribution types for the input parameters, Hong and Roh[48].

In recent years advanced methods such as Artificial neural network has been incorporated in reliability approach by Cho[49]. Results of this study showed that the choice of finite element, finite difference or limit equilibrium methods does not affect the results of reliability approach while using ANN based response surface model.

One more advanced optimization technique named 'Harmony search meta-heuristic algorithm' has been successfully implied in the field of reliability analysis of slopes by Khajehzadeh et al[50]. This technique also gives lower values of reliability index as compared to the traditional methods. The advantage of using this method is its simplicity and programming can be easily done in MATLAB and its ability to construct a new vector from a combination of all existing vectors (i.e. all harmonies in the Harmony Memory) whereas one more similar popular technique genetic algorithm constructs a new vector only from two existing vectors (i.e. the parents). Also, harmony search is independent to consider each component variable in a vector, but the genetic algorithm has to maintain the structure of a gene thus not able to consider each component.

Khajehzadeh[51] applied a modified form of particle swarm optimization method i.e. 'Hybrid chaotic particle swarm optimization with harmony search' to optimize the reliability index function. In comparison of this new method with basic PSO and other optimization techniques it has been concluded that it calculates smaller values of the reliability index and factor of safety and generates superior results in terms of accuracy and convergence rate.

These studies clearly showed that shear strength parameters of soil i.e. cohesion and friction angle have direct relationship with reliability index, as mean of these parameters increases reliability index also increases and as the coefficient of variation of these parameters increases, reliability index decreases, Khajehzadeh[51].

Reliability analysis also depends upon the choice of probability distribution type (normal or lognormal distribution) for random variables, Metya[52]. Normal distribution of random variables gives lesser value of reliability index as compared to log-normal distribution. But the most commonly used technique i.e. mean value first order second moment method does not make the use of information on probability distributions. This difference can be seen in other advanced reliability methods such as First order reliability method, Monte-carlo simulation method.

Thus reliability approach seems to be more useful than deterministic approach in defining stability of a slope. Probability of failure or reliability index gives much useful information about the failure than factor of safety used in deterministic approach. But more soil data is required in reliability approach as distribution graph of each design parameter is used in it instead of a single value. Also an

expertise is required to analyze the results given by reliability approach and to choose a suitable and economic stability design factor for slope.

IV. CONCLUSION

Various available methods for slope stability analysis are discussed in this paper. It can be concluded from the above discussion that LEMs are simple and less accurate than another available deterministic methods like FEM, FDM, limit analysis but these advanced methods need enough time and knowledge to imply.

On the other hand reliability approach is better than deterministic approach in defining the risk and probability of failure of a slope. Due to the reason that reliability index account for parameter variability and uncertainties, they have considerably more spread than factor of safety.

Further it has been seen that modern optimization techniques are very useful in minimizing FOS and reliability index or in locating critical slip surface either deterministic or probabilistic.

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