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STUDY OF A TRANSPORTATION PROBLEM OF AN ESSENTIAL ITEM FROM THE INDIAN ORIGINS TO THE KSA DESTINATIONS USING NQM

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Abstract— **we considered a transportation problem of an essential item, rice, from the different origins of India to different destinations of KSA and formulated the problem as a LPP model. We obtained an IBFS to the problem by VAM and NQM and compared the results and displayed in the tables. The key idea in NQM is to minimize the best combinations of the solution to reach the optimal solution. Comparatively, applying the NQM in the proposed method obtains the best initial feasible solution to the transportation problem and performs faster than the existing methods with a minimal computation time and less complexity. The proposed method is therefore an attractive alternative to traditional problem solution methods. The NQM can be used successfully to solve all different transportation problems in different areas of study. Finally, MODI method has applied for optimal solution for the effectiveness of the proposed method.**

Keywords— Transportation Models, Initial Basic Feasible Solution, Optimal Solution, NQM and VAM.

I. INTRODUCTION

Industries require planning in transporting their products from production centers to the users end with minimal transporting cost to maximize profit. This process is known as Transportation Problem which is used to analyze and minimize transportation cost. This problem is well discussed in operation research for its wide application in various fields, such as scheduling, personnel assignment, product mix problems and many others, so that this problem is really not confined to transportation or distribution only. In the solution procedure of a transportation problem, finding an initial basic feasible solution is the prerequisite to obtain the optimal solution. Again, development is a continuous and endless process to find the best among the bests. The growing complexity of management calls for development of sound methods and techniques for solution of the problems. Considering these factors, this research aims to propose an algorithm "NQM" to obtain an initial basic feasible solution for the transportation problems. Several numbers of numerical problems are also solved to justify the method. Obtained results show that the proposed algorithm is effective in solving transportation problems.

The Transportation problem is one of the traditional function of the Linear Programming Problems. Transportation model provides a greater impact on the transportation of the commodities from the manufacturing places [5]. The basic Transportation problem was initially proposed by Hitch Cock [3][6] Transportation problem is being prioritized in both service and manufacturing industries. It is a network optimization problem which is well known in operation research. This problem is not only used in transportation problem but also in various fields like scheduling, personnel assignment, product mix problems, etc. Now a day's transportation problem has become a standard application for industrial organizations having several manufacturing units, warehouses and distribution centers. Following figure: 1 shows the transportation network.

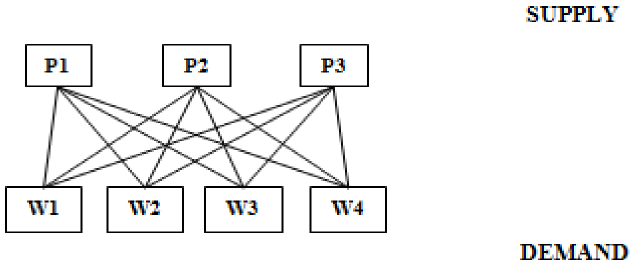

Figure: 1 - Network of T.P.

A Transportation problem is one of the earliest and most important applications of LPP. Description of a classical transportation problem can be given as follows. A certain amount of homogeneous commodity is available at number of sources/origins and a fixed amount is required to meet the demand at each number of destinations/distribution centers. Then finding an optimal schedule of shipment of the commodity with the satisfaction of demands at each destination is the main goal of the problem. In 1941, Hitchcock [1] developed the basic transportation problem along with the constructive method of solution and later in 1949 Koopmans [2] discussed the problem in detail. Again in 1951 Dantzig [3] formulated the transportation problem as LPP and also provided the solution method. The transportation problem in which the objective is to minimize the total cost of shipping a single commodity from a number of sources *(m)* to a number of destinations or sinks *(n)* Because of the special structure of the transportation problem, a special algorithm can be designed to find an optimal solution efficiently. The most important and successful applications in the optimization refers to transportation problem (TP).

The basic steps for obtaining an optimum solution to a transportation problem are: (P. K. Gupta and Man Mohan, (2003) (11).
Step 1: Mathematical model of the problem
Step 2: Finding an initial basic feasible solution (IBFS)
Step 3: To test whether the solution is an optimal one or not. If not, improve it further till the optimality is achieved.

Most of the time the initial basic feasible solution of transportation problem is calculated by using the methods of North West Corner Method or Least-Cost Method or Vogel's Approximation Method, and then finally the optimality is checked by MODI (modified distribution method). Many researchers have provided improved or different algorithm to solve transportation problem. In last few years Hakim, M. A. (2012) [5], Khan, A. R. (2011) [6, 14], Azad, S. M. A. K. et al. (2017) [7], Ahmed, M. M., et. al (2016) [8], Raigar, S. and Modi, D. G. (2017) [9], Hosseini, E. (2017) [10] developed new methods to determine initial basic feasible solution of transportation problem.

In this paper, we considered a transportation problem of an essential item, rice, from the different origins of India to different destinations of KSA and formulated the problem as a LPP model. We obtained an IBFS to the problem by VAM and NQM and compared the results and displayed in the tables. The key idea in NQM is to minimize the combinations of the solution by choosing the best cells to reach the optimal solution. Comparatively, applying the NQM in the proposed method obtains the best initial feasible solution to a transportation problem and performs faster than the existing methods with a minimal computation time and less complexity. The proposed method is therefore an attractive alternative to traditional problem solution methods. The NQM can be used successfully to solve all different transportation problems in different areas of study. Finally, MODI method has applied for optimal solution for the effectiveness of the proposed method.

II. MATHEMATICAL MODEL OF THE PROBLEM

In this problem goods are transported from a set of sources to a set of destinations subject to the supply and demand of the source and destination, respectively, such that the total cost of transportation is minimized (Ahmed, M.M., et. al 2016) [8, 14].

Consider a homogeneous commodity, rice, is transported from the different origins (O_1, O_2, O_3 and O_4) of India to the different destinations (D_1, D_2, D_3 and D_4) of KSA. Let a_i represents the amount of the rice is available at ith origin with a_1 = 2200 quintals, a_2 = 2000 quintals, a_3 = 1800 quintals and a_4 = 2400 quintals. Similarly, Let b_j represents the amount of the rice is required at jth destination with b_1 = 1600 quintals, b_2 = 2500 quintals, b_3 = 1900 quintals and b_4 = 2600 quintals. The transportation problem has shown below table.1.

Table. 1: Transportation problem

	D1	D2	D3	D4	Supply (a_i)
O1	10	30	25	15	2200
O2	20	15	20	10	2000
O3	10	30	20	20	1800
O_4	30	40	35	45	2400
Demand(b_j)	1600	2300	1900	2600	

Let c_{ij} be the unit transportation cost in SAR per a quintal of rice from O_i to D_j, the requirements of the destinations D_j, j = 1, 2, 3, 4, must be satisfied by the supply of available units at the points of origin O_i, i = 1, 2, 3, 4. if x_{ij} is the number of units that are shipped from O_i to D_j, then the problem in determining the values of the variables x_{ij}, i = 1, 2, 3, 4 and j = 1, 2, 3, 4, should minimize the total of the transportation/shipping costs.

The main objective of transportation problem is to determine the amount of the commodity x_{ij} transported from the origins to the destinations so as to minimize the total transportation cost.

The total transportation cost (Z) in the above problem is

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Mini. $Z = 10x_{11} + 30x_{12} + 25x_{13} + 15x_{14} + 20x_{21} + 15x_{22} + 20x_{23} + 10x_{24} + 10x_{31} + 30x_{32} + 20x_{33} + 20x_{34} + 30x_{41} + 40x_{42} + 35x_{43} + 45x_{44}$.

Subject to the constraints,

$$x_{11} + x_{12} + x_{13} + x_{14} = 2200$$
$$x_{21} + x_{22} + x_{23} + x_{24} = 2000$$
$$x_{31} + x_{32} + x_{33} + x_{34} = 1800$$
$$x_{41} + x_{42} + x_{43} + x_{44} = 2400$$
$$x_{11} + x_{21} + x_{31} + x_{41} = 1600$$
$$x_{12} + x_{22} + x_{32} + x_{42} = 2300$$
$$x_{13} + x_{23} + x_{33} + x_{43} = 1900$$
$$x_{14} + x_{24} + x_{34} + x_{44} = 2600$$
$$x_{ij} \geq 0,\ i = 1, 2, 3, 4 \text{ and } j = 1, 2, 3, 4.$$

Mathematically, the transportation problem can be represented as a linear programming model. Since the objective function and the constraints are linear functions and that can adopt the linear programming (LP) technique with equality constraints.

LP technique can be used in different product areas such as oil plum industry [4, 13]. However, The LP technique can be generally used by genetic algorithm such as Sudha at el. article [5, 12]. The transportation solution problem can be found with a good success in the improving the service quality of the public transport systems [6, 11]. As well as, the transportation solution problem is used in the electronic commerce where the area of globalization the degree of competition in the market article [7, 10], and many other fields.

However, there are several different algorithms to solve transportation problem that represented as LP model. Among these are the known algebraic procedures of the simplex method, which may not be the best method to solve the problem. Therefore, more efficient and simpler procedures have been improved to solve transportation problems. Typically, the standard scenario for solving transportation problems is working by sending units of a product across a network of highways that connect a given set of cities. Each city is considered as a source (S) in that units will be shipped out from, while units are demanded there when the city is considered as a sink (D). In this scenario, each sink has a given demand, the source has a given supply, and the airway that connects source with sink as a pair has a given transportation cost/(shipment unit).

The problem is to determine an optimal transportation scheme that is to minimize the total of the shipments cost between the nodes in the network model, subject to supply and demand constraints. As well as, this structure arises in many applications such as; the sources represent warehouses and the sinks represent retail outlets. Moreover, Ad-hoc networks are designed dynamically by group of mobile devices. In Ad-hoc network, nodes between source and destination act as a routers so that source node can communicate with the destination node [8,9].

III. IBFS USING VAM

There are several methods for finding an initial basic feasible solution of the transportation problems which are based on different of special linear programming methods, among these are: Northwest Corner method (NCM), Least cost method (LCM), Vogel's approximation method (VAM), Row Minimum Method (RMM), Column Minimum Method (CMM) etc.. Basically, these methods are different in term of the quality for the produced basic starting solution and the best starting solution that yields minimize the objective function value. Vogel's Approximation Method (VAM) is one of the conventional methods that gives better Initial Basic Feasible Solution (IBFS) of a Transportation Problem (TP). This method considers the row penalty and column penalty of a Transportation Table (TT) which are the differences between the lowest and next lowest cost of each row and each column of the TT respectively. In this study, we used the Vogel's approximation method, since it generally produces better starting solutions than other solving methods. Following table shows the IBFS by using VAM.

Table. 2: IBFS by using VAM.

	D₁		**D₂**		**D₃**		**D₄**		**a_i**
O_1	1600	10		30		25	600	15	2200
O_2		20	2000	15		20		10	2000
O_3		10		30		20	1800	20	1800
O_4		30	300	40	1900	35	200	45	2400
b_j	1600		2300		1900		2600		

Total transportation cost by using VAM is, 1600*10 + 600*15 +2000*15 + 1800*20 + 300*40 + 1900*35+ 200*45 = SAR 178, 500.

IV. IBFS USING NEW QUICK METHOD (NQM):

Step1: We must check the matrix balanced or not, If the total supply is equal to the total demand, then the matrix is balanced and also apply Step 2. If the total supply is not equal to the total demand, then we add a dummy row or column as needed to make supply is equal to the demand. So the transportation costs in this row or column will be assigned to zero.

Table. 3: Balanced Transportation problem.

	D1	D2	D3	D4	Supply (a_i)
O1	10	30	25	15	2200
O2	20	15	20	10	2000
O3	10	30	20	20	1800

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O_4	30	40	35	45	2400
Demand (b_j)	1600	2300	1900	2600	8400

Step2: Select the best cells, that is for minimizing problems to the minimum cost, and maximizing profit to the maximum cost. Therefore, this step can be done by electing the best two cells in each row. If the cell repeated more than two times, then the cell should be elected again. As well as ,the columns must be checked such that if it is not have cells so that the cells will be elected for them. However, if the cell is repeated more than one time, then elect it again. We determine the best combination that will produce the lowest transportation costs, where is one cell for each row and column. The result from this shown in (Table 4).

Table. 4: Selection of less cost cells in rows.

	D1	D2	D3	D4	Supply (a_i)
O1	10	30	25	15	2200
O2	20	15	20	10	2000
O3	10	30	20	20	1800
O_4	30	40	35	45	2400
Demand (b_j)	1600	2300	1900	2600	8400

Step3: Find the combinations by determining one cell for each row and column, this should be done by starting from the row that have the least cells, and then delete that row and column. If there is situation that have no cell for some rows or columns, then directly elect the best available cell. Repeat Step 3 by determining the next cell in the row that started from. Compute and compare the summation of cells for each combination. This is to determine the best combinations. We identify the row with the smallest cost cell from the chosen combination. Then we allocate the supply and the demand to the variable with the least unit cost in the selected row and column. Also, we should adjust the supply and the demand by crossing out the row/column to be then assigned to zero (Table 5). If the column is assigned to zero, then we elect the cost in the cell (d3,s3) because it has a lower cost than the determined cell (d3,s4) from the chosen combination (Table 6).

Table. 5: Selection of less cost cells in columns.

	D1	D2	D3	D4	Supply (a_i)
O1	10	30	25	15	2200
O2	20	15	20	10	2000
O3	10	30	20	20	1800
O_4	30	40	35	45	2400

Table. 6: Allotment of the commodities in the cells.

	D_1		D_2		D_3		D_4		a_i
O_1	1600	10		30		25	600	15	2200
O_2		20		15		20	2000	10	2000
O_3		10	1000	30	800	20		20	1800
O_4		30	1300	40	100	35		45	2400
b_j	1600		2300		1900		2600		8400

Step4: Identify the row with the smallest cost from the chosen combination. Then allocate the demand and the supply as much as possible to the variable with the least unit cost in the selected row or column. Also, we should adjust the supply and demand by crossing out the row/column to be then assigned to zero. If the row or column is not assigned to zero, then we check the selected row if it has an element with lowest cost comparing to the determined element in the chosen combination, then we elect it.

Step5: Elect the next least cost from the chosen combination and repeat Step 4 until all columns and rows are satisfied. The final table of the problem is as follows,

Table. 7: Selection of best allocations in the cells.

	D_1		D_2		D_3		D_4		a_i
O_1	1600	10		30		25	600		2200
O_2		20		15		20	2000	10	2000
O_3		10		30	1800	20		20	1800

O_4		30	2300		100	35		45	2400
b_j	1600		2300		1900		2600		8400

The total transportation cost by using NQM is, 1600*10 + 600*15 +2000*10 + 1800*20 + 2300*40 + 100*35 = SAR 1,77,500.
which is better IBFS comparatively with the solution obtained by VAM.

V. MOVING TOWARDS OPTIMALITY

In order to verify whether the above IBFS using NQM is optimal solution or not, we can apply MODI (Modified Distribution method) or u-v method as follows:
Step-1: Determine an initial basic feasible solution as we derived in the above problem by New Quick Method. Then take the costs only of the occupied cells which are basic cells and corresponding variables are basic variables. If the number of basic cells is equal to m+n-1, then IBFS is non-degenarate. Otherwise degenerate, make degenerate as non-degenerate by assigning small number ε in the required number of unoccupied cells.
Step-2: Set v1, v2, v3…etc. against the corresponding column and set u1, u2, u3…etc. against the corresponding row. Then determine a set of ui and vj such that for each occupied cell, using ui+ vj = cij.
Step-3: Assign 0 to one of the ui or vj for which the corresponding row or column have the maximum number of individual allocations.
Step-4: Then find the cell evaluations (Subtract the above matrix cells from the corresponding cells of original matrix) ui+vj for each unoccupied cell and enter at the middle of the corresponding unoccupied cell and encircle.
Step-5: Then we calculate dij(difference of each occupied cell) by the dij= Cij– (ui+vj) for each unoccupied cell and enter at the middle of the corresponding unoccupied cell and encircle.
Step-6: If all the dij is non-negative, then the basic feasible solution is optimal. On the other hand, if anyone of dij is –ve, then the basic feasible solution is not optimal.
Step-7: Select the largest negative value of dij. If there have more than one equal cell, then any one can be chosen. Then draw a closed loop for the unoccupied cell. Starting the closed loop with the largest negative value of dij and draw a closed loop with the occupied cells only.
Step-8: Mark the identified cell as +ve and each occupied cell at the corners of the path alternatively –ve , +ve , -ve, . . and so on.
Step-9: check all the negative position and consider the smallest transportation cost that has been assigned a – ve sign. Now, + and –demand and supply values of all the positions of + and -.
Step-10: Repeat the whole procedure until the optimum solution is obtained.

The optimal solution of the problem using MODI method is, x_{14} = 2200, x_{22} = 1600, x_{24} = 400, x_{31} = 1600, x_{33} = 200, x_{42} = 700, x_{43} =1700.
Hence, the minimum transportation cost of the problem is, 2200*15 + 1600*15 + 400*10 + 1600*10 + 200*20 + 700*40 + 1700*35 = SAR 1,68,500.
We solved some more TP along with the original one and kept the results in the below table to see the effectiveness of the NQM.

TP	VAM (In SAR)	NQM (In SAR)	MODI (In SAR)
4X4 matrix	178,500	177,500	168,500
3X4 matrix	220,000	200,500	200,500
4X3 matrix	112,800	100,000	95,000
4X4 matrix	2,200	1,850	1,850
5X4 matrix	164	155	150

VI. CONCLUSION

In this study, we proposed NQM for finding the IBFS to the transportation problem. It refers to choose the best distribution of cost and time from the all combinations. The NQM obtained the optimal solution or the closest to optimal solution with a minimum computation time. As well as, use of NQM reduces the complexity of the problems which is shown in the tables.

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