



A NOTE ON INTUITIONISTIC FUZZY $\hat{\beta}$ GENERALIZED IRRESOLUTE MAPPINGS

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ABSTRACT: In this paper a new class of mapping called intuitionistic fuzzy $\hat{\beta}$ generalized irresolute mapping in intuitionistic fuzzy topological space is introduced and its properties are studied. Further the notion of intuitionistic fuzzy $\hat{\beta}_a T_{1/2}$ space and intuitionistic fuzzy $\hat{\beta}_b T_{1/2}$ are introduced.

KEYWORDS: Intuitionistic fuzzy topology, intuitionistic fuzzy $\hat{\beta}$ generalized closed set, intuitionistic fuzzy $\hat{\beta}$ generalized open set, intuitionistic fuzzy $\hat{\beta}$ generalized irresolute mapping, intuitionistic fuzzy contra $\hat{\beta}$ generalized irresolute mapping, intuitionistic fuzzy $\hat{\beta}_a T_{1/2}$ space and intuitionistic fuzzy $\hat{\beta}_b T_{1/2}$ space.

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I. INTRODUCTION

After the introduction of Fuzzy set (FS) by Zadeh [12] in 1965 and fuzzy topology by Chang [3] in 1967, several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. The concept of intuitionistic fuzzy set (IFS) was introduced by Atanassov in 1983 as a generalization of fuzzy sets. In 1997 Coker [4] introduced the concept of intuitionistic fuzzy topological space. In this paper we introduce the notion of intuitionistic fuzzy $\hat{\beta}$ generalized irresolute mappings, intuitionistic fuzzy contra $\hat{\beta}$ generalized irresolute mapping in intuitionistic fuzzy topological space and studied some of their properties. We provide some characterizations of intuitionistic fuzzy $\hat{\beta}$ generalized irresolute mappings, intuitionistic fuzzy contra $\hat{\beta}$ generalized irresolute mappings and established the relationships with other classes of early defined forms of intuitionistic fuzzy mappings.

II. PRELIMINARIES

Definition 2.1: [1] Let X be a non empty fixed set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$$

where the functions $\mu_A(x): X \rightarrow [0, 1]$ and $\nu_A(x): X \rightarrow [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A , respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote the set of all intuitionistic fuzzy sets in X by $\text{IFS}(X)$.

Definition 2.2: [1] Let A and B be IFSs of the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ and $B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle / x \in X \}$. Then

- (a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$
- (b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$
- (c) $A^c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle / x \in X \}$
- (d) $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle / x \in X \}$
- (e) $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle / x \in X \}$

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A \rangle$ instead of $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$. Also for the sake of simplicity, we shall use the notation $A = \{ \langle x, (\mu_A, \mu_B), (\nu_A, \nu_B) \rangle \}$ instead of $A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B) \rangle$.

The intuitionistic fuzzy sets $0 \sim = \{ \langle x, 0, 1 \rangle / x \in X \}$ and $1 \sim = \{ \langle x, 1, 0 \rangle / x \in X \}$ are respectively the empty set and the whole set of X .

Definition 2.3: [3] An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms.

- (i) $0 \sim, 1 \sim \in \tau$
- (ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$
- (iii) $\cup G_i \in \tau$ for any family $\{ G_i / i \in J \} \subseteq \tau$.



In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X .
 The complement A^c of an IFOS A in IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X .

Definition 2.4:[3] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X . Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by
 $\text{int}(A) = \cup \{ G / G \text{ is an IFOS in } X \text{ and } G \subseteq A \}$,
 $\text{cl}(A) = \cap \{ K / K \text{ is an IFCS in } X \text{ and } A \subseteq K \}$
 Note that for any IFS A in (X, τ) , we have $\text{cl}(A^c) = [\text{int}(A)]^c$ and $\text{int}(A^c) = [\text{cl}(A)]^c$.

Definition 2.5: [4] An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be a
 (i) intuitionistic fuzzy semi closed set (IFSCS for short) if $\text{int}(\text{cl}(A)) \subseteq A$,
 (ii) intuitionistic fuzzy pre-closed set (IFPCS for short) if $\text{cl}(\text{int}(A)) \subseteq A$,
 (iii) intuitionistic fuzzy α -closed set (IF α CS for short) if $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$,
 (iv) intuitionistic fuzzy γ -closed set (IF γ CS for short) if $\text{cl}(\text{int}(A)) \cap \text{int}(\text{cl}(A)) \subseteq A$
 The respective complements of the above IFCSs are called their respective IFOSs.

The family of all IFSCSs, IFPCSs, IF α CSs and IF γ CSs (respectively IFOSOs, IFPOSo, IF α OSs and IF γ OSs) of an IFTS (X, τ) are respectively denoted by IFSC(X), IFPC(X), IF α C(X) and IF γ C(X) (respectively IFSO(X), IFPO(X), IF α O(X) and IF γ O(X)).

Definition 2.6:[11] Let A be an IFS in an IFTS (X, τ) . Then
 $\text{sint}(A) = \cup \{ G / G \text{ is an IFOS in } X \text{ and } G \subseteq A \}$,
 $\text{scl}(A) = \cap \{ K / K \text{ is an IFCS in } X \text{ and } A \subseteq K \}$.
 Note that for any IFS A in (X, τ) , we have $\text{scl}(A^c) = (\text{sint}(A))^c$ and $\text{sint}(A^c) = (\text{scl}(A))^c$.

Definition 2.7:[10] An IFS A in an IFTS (X, τ) is an intuitionistic fuzzy generalized closed set (IFGCS in short) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X .

Definition 2.8:[10] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy generalized semi closed set (IFGSCS in short) if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ) .

Definition 2.9:[10] An IFS A is said to be an intuitionistic fuzzy generalized semi open set (IFGSOS in short) in X if the complement A^c is an IFGSCS in X .
 The family of all IFGSCSs (IFGSOSs) of an IFTS (X, τ) is denoted by IFGSC(X) (IFGSO(X)).

Definition 2.10:[5] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be intuitionistic fuzzy

continuous (IF continuous in short) if $f^{-1}(B) \in \text{IFO}(X)$ for every $B \in \sigma$.

Definition 2.11: [5] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be an

- (i) intuitionistic fuzzy semi continuous mapping (IFS continuous mapping for short) if $f^{-1}(B) \in \text{IFSO}(X)$ for every $B \in \sigma$
- (ii) intuitionistic fuzzy α -continuous mapping (IF α continuous mapping for short) if $f^{-1}(B) \in \text{IF}\alpha\text{O}(X)$ for every $B \in \sigma$
- (iii) intuitionistic fuzzy pre continuous mapping (IFP continuous mapping for short) if $f^{-1}(B) \in \text{IFPO}(X)$ for every $B \in \sigma$
- (iv) intuitionistic fuzzy γ continuous mapping (IF γ continuous mapping for short) if $f^{-1}(B) \in \text{IF}\gamma\text{O}(X)$ for every $B \in \sigma$.

Definition 2.12: [11] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be an intuitionistic fuzzy generalized continuous mapping (IFG continuous mapping for short) if $f^{-1}(B) \in \text{IFGC}(X)$ for every IFCS B in Y .

Definition 2.13: [11] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be an intuitionistic fuzzy semi-pre continuous mapping (IFSP continuous mapping for short) if $f^{-1}(B) \in \text{IFSP}\text{O}(X)$ for every $B \in \sigma$.

Result 2.14:[8] Every IF continuous mapping is an IFG continuous mapping.

Definition 2.15:[8] A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is called an intuitionistic fuzzy generalized semi continuous (IFGS continuous in short) if $f^{-1}(B)$ is an IFGSCS in (X, τ) for every IFCS B of (Y, σ) .

Definition 2.16: [8] An IFTS (X, τ) is said to be an intuitionistic fuzzy $\hat{\beta}_a T_{1/2}$ (IF $\hat{\beta}_a T_{1/2}$ in short) space if every IF $\hat{\beta}_a$ GCS in X is an IFCS in X .

Definition 2.17: [8] An IFTS (X, τ) is said to be an intuitionistic fuzzy $\hat{\beta}_b T_{1/2}$ (IF $\hat{\beta}_b T_{1/2}$ in short) space if every IF $\hat{\beta}_b$ GCS in X is an IFGCS in X .

Definition 2.18:[9] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be an intuitionistic fuzzy irresolute (IF irresolute in short) if $f^{-1}(B) \in \text{IFCS}(X)$ for every IFCS B in Y .

Definition 2.19:[9] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be an intuitionistic fuzzy generalized irresolute (IFG irresolute in short) if $f^{-1}(B) \in \text{IFGCS}(X)$ for every IFGCS B in Y .



III. INTUITIONISTIC FUZZY GENERALIZED IRRESOLUTE MAPPINGS

In this section, we have introduced intuitionistic fuzzy generalized irresolute mappings and studied some of their properties.

Definition 3.1: A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is called an intuitionistic fuzzy generalized irresolute (IF β G irresolute) mapping if $f^{-1}(A)$ is an IF β GCS in (X, τ) for every IF β GCS A of (Y, σ) .

Theorem 3.2: If $f : (X, \tau) \rightarrow (Y, \sigma)$ is an IF β G irresolute mapping, then f is an IF β G continuous mapping but not conversely.

Proof: Assume that f is an IF β G irresolute mapping. Let A be any IFCS in Y . Since every IFCS is an IF β GCS, A is an IF β GCS in Y . By hypothesis $f^{-1}(A)$ is an IF β GCS in X . Hence f is an IF β G continuous mapping.

Example 3.3: Let $X = \{ a, b \}$, $Y = \{ u, v \}$ and $G_1 = \langle x, (0.1, 0.3), (0.6, 0.3) \rangle$, $G_2 = \langle y, (0.3, 0.1), (0.5, 0.6) \rangle$. Then $\tau = \{ 0., G_1, 1. \}$ and $\sigma = \{ 0., G_2, 1. \}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IF β G continuous mapping. Let $B = \langle y, (0.1, 0.1), (0.6, 0.4) \rangle$ is an IF GCS in Y . But $f^{-1}(B) = \langle x, (0.1, 0.1), (0.6, 0.4) \rangle$ is not an IF GCS in X . Therefore, f is not an IF G irresolute mapping.

Theorem 3.4: If $f : (X, \tau) \rightarrow (Y, \sigma)$ is an IF β G irresolute mapping, then f is an IFGS continuous mapping but not conversely.

Proof: Assume that f is an IF β G irresolute mapping. Let A be any IFCS in Y . Since every IFCS is an IF β GCS, A is an IF β GCS in Y . By hypothesis $f^{-1}(A)$ is an IF β GCS in X . This implies $f^{-1}(A)$ is an IFGCS in X . Hence f is an IFGS continuous mapping.

Example 3.5: Let $X = \{ a, b \}$, $Y = \{ u, v \}$ and $G_1 = \langle x, (0.1, 0.3), (0.3, 0.3) \rangle$, $G_2 = \langle y, (0.3, 0.1), (0.4, 0.4) \rangle$. Then $\tau = \{ 0., G_1, 1. \}$ and $\sigma = \{ 0., G_2, 1. \}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IFGS continuous mapping. Let $B = \langle y, (0.1, 0.1), (0.3, 0.4) \rangle$ is an IF β GCS in Y . But $f^{-1}(B) = \langle x, (0.1, 0.1), (0.3, 0.4) \rangle$ is not an IF β GCS in X . Therefore, f is not an IF G irresolute mapping.

Theorem 3.6: If $f : (X, \tau) \rightarrow (Y, \sigma)$ is an IF β G irresolute mapping in an IF β $T_{1/2}$ space X , then f is an IF continuous mapping.

Proof: Let A be an IFCS in Y . Then A is an IF GCS in Y . Since f is an IF β G irresolute, $f^{-1}(A)$ is an IF β GCS in X . Also, since X is an IF β $T_{1/2}$ space, $f^{-1}(A)$ is an IFCS in X . Hence f is an IF continuous mapping.

Theorem 3.7: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \eta)$ be IF β G irresolute mappings. Then $g \circ f : (X, \tau) \rightarrow (Z, \eta)$ is an IF β G irresolute mapping.

Proof: Let A be an IF β GCS in Z . Then by hypothesis, $g^{-1}(A)$ is an IF β GCS in Y . Since f is an IF β G irresolute mapping, $f^{-1}(g^{-1}(A))$ is an IF β GCS in X . Thus, $(g \circ f)^{-1}(A)$ is an IF β GCS in X . Therefore $g \circ f$ is an IF β G irresolute mapping.

Theorem 3.8: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an IF β G irresolute mapping and $g : (Y, \sigma) \rightarrow (Z, \eta)$ be an IF β G continuous mapping. Then $g \circ f : (X, \tau) \rightarrow (Z, \eta)$ is an IF β G continuous mapping.

Proof: Let A be an IFCS in Z . Then by hypothesis, $g^{-1}(A)$ is an IF GCS in Y . Since f is an IF β G irresolute mapping, $f^{-1}(g^{-1}(A))$ is an IF β GCS in X . Thus, $(g \circ f)^{-1}(A)$ is an IF β GCS in X .

Therefore, $g \circ f$ is an IF G continuous mapping.

Theorem 3.9: If $f : (X, \tau) \rightarrow (Y, \sigma)$ is an IF β G irresolute mapping in an IF β $T_{1/2}$ space in X , then f is an IFG irresolute mapping.

Proof: Let A be an IFGCS in Y . Then A is an IF β GCS in Y . Therefore $f^{-1}(A)$ is an IF β GCS in X , by hypothesis. Since X is an IF β $T_{1/2}$ space, $f^{-1}(A)$ is an IFGCS in X . Hence f is an IFG irresolute mapping.

Theorem 3.10: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a mapping from an IFTS X into an IFTS Y . Then the following conditions are equivalent if X and Y are IF β $T_{1/2}$ spaces:

- (i) f is an IF β G irresolute mapping
- (ii) $f^{-1}(B)$ is an IF β GOS in X for each IF β GOS in Y
- (iii) $cl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$ for each IFS B of Y .

Proof:(i) \Rightarrow (ii): Obviously true.

(ii) \Rightarrow (iii): Let B be any IFS in Y . Clearly $B \subseteq cl(B)$. Then $f^{-1}(B) \subseteq f^{-1}(cl(B))$. Since $cl(B)$ is an IFCS in Y , $cl(B)$ is an IF β GCS in Y . Therefore, $f^{-1}(cl(B))$ is an IF β GCS in X , by hypothesis. Since X is an IF β $T_{1/2}$ space, $f^{-1}(cl(B))$ is an IFCS



in X . Hence $\text{cl}(f^{-1}(B)) \subseteq \text{cl}(f^{-1}(\text{cl}(B))) = f^{-1}(\text{cl}(B))$. That is $\text{cl}(f^{-1}(B)) \subseteq f^{-1}(\text{cl}(B))$.

(iii) \Rightarrow (i): Let B be an $\text{IF}\hat{\beta}\text{GCS}$ in Y . Since Y is an $\text{IF}\hat{\beta}_a T_{1/2}$ space, B is an IFCS in Y and $\text{cl}(B) = B$. Hence $f^{-1}(B) = f^{-1}(\text{cl}(B)) \supseteq \text{cl}(f^{-1}(B))$, by hypothesis. But clearly $f^{-1}(B) \subseteq \text{cl}(f^{-1}(B))$. Therefore, $\text{cl}(f^{-1}(B)) = f^{-1}(B)$. This implies $f^{-1}(B)$ is an IFCS in X and hence it is an $\text{IF}\hat{\beta}\text{GCS}$ in X . Thus f is an $\text{IF}\hat{\beta}\text{G}$ irresolute mapping.

IV. INTUITIONISTIC FUZZY CONTRA GENERALIZED IRRESOLUTE MAPPINGS

In this section, we have introduced intuitionistic fuzzy contra $\hat{\beta}$ generalized irresolute mapping and studied some of its properties.

Definition 4.1: A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is called an intuitionistic fuzzy contra $\hat{\beta}$ generalized irresolute (IFC $\hat{\beta}\text{G}$ irresolute in short) mapping if $f^{-1}(A)$ is an $\text{IF}\hat{\beta}\text{GCS}$ in (X, τ) for every $\text{IF}\hat{\beta}\text{GOS}$ A of (Y, σ) .

Theorem 4.2: If $f : (X, \tau) \rightarrow (Y, \sigma)$ is an IFC $\hat{\beta}\text{G}$ irresolute mapping, then f is an IFC $\hat{\beta}\text{G}$ continuous mapping but not conversely.

Proof: Let f be an IFC $\hat{\beta}\text{G}$ irresolute mapping. Let A be any IFCS in Y . Since every IFCS is an $\text{IF}\hat{\beta}\text{GCS}$, A is an $\text{IF}\hat{\beta}\text{GCS}$ in Y . By hypothesis $f^{-1}(A)$ is an $\text{IF}\hat{\beta}\text{GOS}$ in X . Hence f is an IFC $\hat{\beta}\text{G}$ continuous mapping.

Example 4.3: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \langle x, (0.1, 0.3), (0.6, 0.3) \rangle$, $G_2 = \langle y, (0.5, 0.6), (0.3, 0.1) \rangle$. Then $\tau = \{0_-, G_1, 1_-\}$ and $\sigma = \{0_-, G_2, 1_-\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IFC $\hat{\beta}\text{G}$ continuous mapping but f is not an IFC $\hat{\beta}\text{G}$ irresolute mapping, since $B = \langle y, (0.1, 0.1), (0.6, 0.4) \rangle$ is an $\text{IF}\hat{\beta}\text{GOS}$ in Y but $f^{-1}(B) = \langle x, (0.1, 0.1), (0.6, 0.4) \rangle$ is not an $\text{IF}\hat{\beta}\text{GCS}$ in X .

Theorem 4.4: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an IFC $\hat{\beta}\text{G}$ irresolute mapping, then $f^{-1}(A)$ is an IFGSCS in X for every IFOS A in Y but not conversely.

Proof: Let f be an IFC $\hat{\beta}\text{G}$ irresolute mapping. Let A be any IFOS in Y . Since every IFOS is an $\text{IF}\hat{\beta}\text{GOS}$, A is an $\text{IF}\hat{\beta}\text{GOS}$ in Y . By hypothesis, $f^{-1}(A)$ is an $\text{IF}\hat{\beta}\text{GCS}$ in X . This implies $f^{-1}(A)$ is an IFGSCS in X .

Example 4.5: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \langle x, (0.1, 0.3), (0.3, 0.3) \rangle$, $G_2 = \langle y, (0.4, 0.4), (0.3, 0.1) \rangle$. Then $\tau = \{0_-, G_1, 1_-\}$ and $\sigma = \{0_-, G_2, 1_-\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then $f^{-1}(A)$ is an IFGSCS in X for every IFOS A in Y . But f is not an IFC $\hat{\beta}\text{G}$ irresolute mapping, since $B = \langle y, (0.3, 0.4), (0.1, 0.1) \rangle$ is an $\text{IF}\hat{\beta}\text{GOS}$ in Y but $f^{-1}(B) = \langle x, (0.3, 0.4), (0.1, 0.1) \rangle$ is not an $\text{IF}\hat{\beta}\text{GCS}$ in X .

Theorem 4.6: If $f : (X, \tau) \rightarrow (Y, \sigma)$ is an IFC $\hat{\beta}\text{G}$ irresolute mapping, then f is an $\text{IF}\hat{\beta}$ contra continuous mapping if X is an $\text{IF}\hat{\beta}_a T_{1/2}$ space.

Proof: Let A be an IFCS in Y . Then A is an $\text{IF}\hat{\beta}\text{GCS}$ in Y . Therefore $f^{-1}(A)$ is an $\text{IF}\hat{\beta}\text{GOS}$ in X , by hypothesis. Since X is an $\text{IF}\hat{\beta}_a T_{1/2}$ space, $f^{-1}(A)$ is an IFOS in X . Hence f is an $\text{IF}\hat{\beta}$ contra continuous mapping.

Theorem 4.7: If $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \eta)$ are IFC $\hat{\beta}\text{G}$ irresolute mappings, then $g \circ f : (X, \tau) \rightarrow (Z, \eta)$ is an $\text{IF}\hat{\beta}\text{G}$ irresolute mapping.

Proof: Let A be an $\text{IF}\hat{\beta}\text{GCS}$ in Z . Then $g^{-1}(A)$ is an $\text{IF}\hat{\beta}\text{GOS}$ in Y . Since f is an IFC $\hat{\beta}\text{G}$ irresolute mapping, $f^{-1}(g^{-1}(A))$ is an $\text{IF}\hat{\beta}\text{GCS}$ in X . That is $(g \circ f)^{-1}(A)$ is an $\text{IF}\hat{\beta}\text{GCS}$ in X . Hence $g \circ f$ is an $\text{IF}\hat{\beta}\text{G}$ irresolute mapping.

Theorem 4.8: If $f : (X, \tau) \rightarrow (Y, \sigma)$ is an IFC $\hat{\beta}\text{G}$ irresolute mapping and $g : (Y, \sigma) \rightarrow (Z, \eta)$ is an IFC $\hat{\beta}\text{G}$ continuous mapping, then $g \circ f : (X, \tau) \rightarrow (Z, \eta)$ is an $\text{IF}\hat{\beta}\text{G}$ continuous mapping.

Proof: Let A be an IFCS in Z . Then by hypothesis, $g^{-1}(A)$ is an $\text{IF}\hat{\beta}\text{GOS}$ in Y . Since f is an IFC $\hat{\beta}\text{G}$ irresolute mapping, $f^{-1}(g^{-1}(A))$ is an $\text{IF}\hat{\beta}\text{GCS}$ in X . That is $(g \circ f)^{-1}(A)$ is an $\text{IF}\hat{\beta}\text{GCS}$ in X . Hence $g \circ f$ is an $\text{IF}\hat{\beta}\text{G}$ continuous mapping.

Theorem 4.9: If $f : (X, \tau) \rightarrow (Y, \sigma)$ is an $\text{IF}\hat{\beta}\text{G}$ irresolute mapping and $g : (Y, \sigma) \rightarrow (Z, \eta)$ is an IFC $\hat{\beta}\text{G}$ continuous mapping, then $g \circ f : (X, \tau) \rightarrow (Z, \eta)$ is an IFC $\hat{\beta}\text{G}$ continuous mapping.

Proof: Let A be an IFCS in Z . Then by hypothesis $g^{-1}(A)$ is an $\text{IF}\hat{\beta}\text{GOS}$ in Y . Since f is an $\text{IF}\hat{\beta}\text{G}$ irresolute mapping, $f^{-1}(g^{-1}(A))$ is an $\text{IF}\hat{\beta}\text{GOS}$ in X . That is $(g \circ f)^{-1}(A)$ is an $\text{IF}\hat{\beta}\text{GOS}$ in X . Hence $g \circ f$ is an IFC $\hat{\beta}\text{G}$ continuous mapping.

Theorem 4.10: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \eta)$ be any two mappings. If the mapping $g \circ f$ is an IFC $\hat{\beta}\text{G}$ irresolute mapping and X is an $\text{IF}\hat{\beta}_a T_{1/2}$ space, then

- (i) $(g \circ f)^{-1}(B)$ is an $\text{IF}\hat{\beta}\text{GOS}$ in X for each $\text{IF}\hat{\beta}\text{GCS}$ in Z



(ii) $cl((g \circ f)^{-1}(int(B))) \subseteq (g \circ f)^{-1}(B)$ for each IFS B of Z.

Proof:(i): Let B be an $IF_{\beta}^{\alpha}GCS$ in Z. Then B^c is an $IF_{\beta}^{\alpha}GOS$ in Z. By hypothesis, B^c is an $IF_{\beta}^{\alpha}GCS$ in X. This implies B is an $IF_{\beta}^{\alpha}GOS$ in X. Hence $(g \circ f)^{-1}(B)$ is an $IF_{\beta}^{\alpha}GOS$ in X.

(ii): Let B be any IFS in Z and $int(B) \subseteq B$. Then $(g \circ f)^{-1}(int(B)) \subseteq (g \circ f)^{-1}(B)$. Since $int(B)$ is an IFOS in Z, $int(B)$ is an $IF_{\beta}^{\alpha}GOS$ in Z. Therefore $(g \circ f)^{-1}(int(B))$ is an $IF_{\beta}^{\alpha}GCS$ in X, by hypothesis. Since X is an $IF_{\beta}^{\alpha}T_{1/2}$ space, $(g \circ f)^{-1}(int(B))$ is an IFCS in X. Hence $cl((g \circ f)^{-1}(int(B))) = (g \circ f)^{-1}(int(B)) \subseteq (g \circ f)^{-1}(B)$. Therefore, $cl((g \circ f)^{-1}(int(B))) \subseteq (g \circ f)^{-1}(B)$ for each IFS B of Z.

V. CONCLUSION

In this paper we introduced the notion of intuitionistic fuzzy β generalized irresolute mappings, intuitionistic fuzzy contra β generalized irresolute mapping in intuitionistic fuzzy topological space and studied some of their properties.

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