MODELLING NAIRA/POUNDS EXCHANGE RATE VOLATILITY: APPLICATION OF ARIMA AND GARCH MODELS

Saminu Umar
Department of Mathematics and Statistics
Umaru Ali Shinkafi Polytechnic, Sokoto, Nigeria

Shehu Sidi Abubakar
Department of Mathematics and Statistics
Umaru Ali Shinkafi Polytechnic, Sokoto, Nigeria

Abdulrashid M. Salihu
Department of Mathematics and Statistics
Umaru Ali Shinkafi Polytechnic, Sokoto, Nigeria

Zayyanu Umar
Department of Agricultural Science
Shehu Shagari Collage of Education, Sokoto, Nigeria

Abstract—**This study aimed at modelling the daily Naira/Pound exchange rate volatility with ARIMA and GARCH type models with daily exchange rate ranging from June 2016 to July 2019 is obtained from Central Bank of Nigeria. The stationarity of the data series was checked using graphical analysis, Augmented Dickey Fuller (ADF) and Phillips-Perron (PP) tests, it was found out that the exchange rate series is not stationary, the return of the series was obtained and found out to be stationary. It was observed that ARIMA (2, 1, 1) and GARCH (1,1) are the optimal with the highest log-likelihood and lowest AIC and BIC.**

Keywords—**Exchange rate, Volatility, Heteroscedasticity, GARCH**

I. INTRODUCTION

Currency serves as a medium of exchange that simplifies transactions between variant individuals interacting in a marketplace. However, transactions between people who live in different countries are more complicated because of the existence of different mediums of exchange. Obafemi (2017) describe exchange rate as the price of one country's currency in relation to another country. It is the price in which one currency is exchange for another. It measures the domestic worth of an economy; in terms of the currencies of most industrialized countries such as United State of America Dollars, British Pound Sterling.

Exchange rate policy has been identified as one of the endogenous factors that can affect the economic performance of a nation. In Nigeria, the management of exchange rate is carried out by the Central Bank of Nigeria following the adoption of the structural adjustment program policy in 1986, the country has moved from a pegged or rigid exchange rate regime to a more flexible regime. Esam, (2017) in his work highlighted that, no exchange rate is 'clean or pure float', that is a situation where the exchange rate is left completely to be determined by market forces of demand and supply but rather the prevailing system is the managed float whereby the monetary authorities intervene periodically in the foreign exchange market of a country in order to attain some strategic objectives.

Time series is the record of outcomes of a variable according to time, the outcomes may be recorded daily, weekly, monthly, quarterly, yearly or at any other specified interval of time. It is known that time series data are volatile. Undoubtedly, daily exchange rate of one currency for another currency forms a time series. David et al, (2016) stress that a time series may be studied in order to obtain an appropriate model which may be necessary for planning purpose. For example, having an idea of the future exchange rate of a currency for another currency could help the country to guide against inflation, determine its balance of payments and formulate feasible economic policies, among other benefits.

The volatility of financial time series appears to change over time; one class of models that have been developed with a feature that accommodates the dynamics of conditional heteroscedasticity is the autoregressive conditional heteroscedasticity (ARCH) proposed by (Engle, 1982), it is widely adopted to describe and forecast time-varying volatility of financial assets, and extended to generalized autoregressive conditional heteroscedasticity (GARCH) by Bollerslev (1986) has been shown to be useful in modeling variance.

Several studies have applied the Box-Jenkins and GARCH models approach to model and forecast time series, some of these studies include; Shahla et al, (2012), the authors studied and modelled Monthly average foreign exchange rates of Pakistan using ARCH and GARCH models they found ARMA (1,2) and GARCH (1,2) to be best models in terms of fitting and forecasting power of the exchange rate return. Bala et al, (2013), examines monthly exchange–rate return for three major currencies in the Nigerian FX markets: The Naira/USD, Euro and BPS using variants of GARCH volatility models and compared estimates from these models, their study reveals that

TGARCH is the best fitting model for Euro, while ARCH and PARCH (1,1) are the best fitting models for BPS return and Naira/USD returns. For the USD return model with volatility breaks, the best fitting model is the IGARCH. Suleiman et al, (2015), explored and developed the best ARIMA and GARCH models to efficiently forecast monthly crude oil price in Nigeria. The result of their empirical analysis reveals ARIMA (3, 1, 1) and GARCH (2, 1) as the best model for forecasting the crude oil price data series. The aim of this study is to develop time series models (ARIMA and GARCH models) for the Naira/Pounds exchange rate.

II. MATERIAL AND METHODS

A. Data –

The data for this study is of the daily official exchange rate of Nigerian naira per Sterling Pounds. The data is obtained from Central Bank of Nigeria (CBN) with sample period ranging from 20/6/2016 to 31/7/2019 consisting of a total of 747 observations. Econometric time series models ARIMA, ARCH and GARCH are applied to model the data.

B. Autoregressive Integrated Moving Average (ARIMA) Models–

A general notation for Autoregressive Integrated Moving Average (ARIMA) popularly known as Box-Jenkins Methodology (G. P. E. Box and G. M. Jenkins (1978)) is (ARIMA (p,d,q)) where p denotes the number of autoregressive terms, q denotes the number of moving average terms and d denotes the number of times a series must be differenced to induce stationarity.

$$\phi(B)(1 - B)^d X_t = \mu + \theta(B)\varepsilon_t \qquad (1)$$

Where $\phi(B)$ and $\theta(B)$ are polynomials of degrees p and q respectively and $(1 - B)d = \nabla^d$ is the differencing operator and represents the order of differencing the series and ε_t is independently and normally distributed with zero mean and constant variance.

C. Autoregressive Autoregressive Conditional Hetroskedasticity (ARCH) Models–

The first model that provides a systematic framework for volatility modeling is the Autoregressive Conditional Heteroskedastic (ARCH) model of Engle (1982), he proposed a model in which the conditional variance of a time series is a function of past shocks; the model provided a rigorous way of empirically investigating issues involving volatility of economic variables.

ARCH (m) model assumes that

$$y_t = \sigma_t \varepsilon_t$$
$$\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \cdots + \alpha_m a_{t-m}^2$$
$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{m} \alpha_1 a_{t-i}^2 \qquad (2)$$

Where $\{\varepsilon_t\}$ is a sequence of independent and identically distributed (iid) random variables with mean zero $[E(\varepsilon_t) = 0]$ and variance 1 $[E(\varepsilon_t^2) = 1]$, i.e. the random variable $\{\varepsilon_t\}$

is a White noise process, $\alpha_0 > 0$, and $\alpha_i \geq 0$ for i>0. The coefficients α_i must satisfy some regularity conditions to ensure that the unconditional variance of α_t is finite.

D. Generelized Autoregressive Autoregressive Conditional Hetroskedasticity (ARCH) Models–

Bollerslev (1986) proposed a useful extension known as the generalized ARCH (GARCH) model. For a log return series y_t, we assume that the mean equation of the process can be adequately described by an ARMA model. Let $a_t = y_t - \mu_t$ be the mean-corrected log return. Then a_t follows a GARCH (m, s) (where m is the order of the GARCH term σ and s is the order of the ARCH term ε_t) model if

$$\sigma_t^2 = a_0 + a_1 \varepsilon_{t-1}^2 + \cdots a_m \varepsilon_{t-m}^2 + \beta_1 \sigma_{t-1}^2 + L + \beta_s \sigma_{t-s}^2$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{m} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{s} \beta_j \sigma_{t-j}^2 \qquad (3)$$

where again $\{\varepsilon_t\}$ is a sequence of iid random variables with mean 0 and variance 1, $\alpha_0 > 0$, $\alpha_i \geq 0, \beta_j \geq 0$, and $\sum_{i=1}^{\max(m,s)} (\alpha_i + \beta_j) < 1$. It is observed that $\alpha_i = 0$ for i> m and $\beta_j = 0$ for j > s. The latter constraint on $\alpha_i + \beta_j$ implies that the unconditional variance of a_t is finite, whereas its conditional variance σ_t^2 evolves over time. As before, ε_t is assumed to be White noise. Equation (3.4) reduces to a pure ARCH (m) model if s = 0.

III. RESULT AND DISCUSSION

Having explored the theory of ARIMA, ARCH and GARCH models in the preceding section, this section provides an empirical analysis of the daily exchange rate data of Nigerian Naira and Serling pounds. Fig.1 shows the graphical analysis of the exchange rate series, it is observed that there is upward and downward trend indicating volatility clustering which suggests the exchange rate is nonstationary. It is important to confirm whether the data is stationary before using the time series models like Autoregressive Integrated Moving Average (ARIMA), Autoregressive Conditional Heteroscedasticity (ARCH), Generalized Autoregressive Conditional Heteroscedasticity (GARCH)
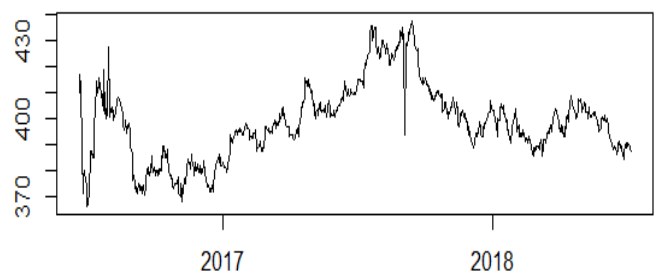

Fig.1: Time series plot of daily exchange rate

The Augmented Dickey Fuller (ADF) and Philip-Perron (PP) tests were performed, the result is presented in t able1, since the P-value for both tests is greater than the critical value, we do not reject the null hypothesis of the presence of unit root in the series and conclude that the exchange rate series

Phillips-Perron Test	-37.467
P-value	0.01

is nonstationary, this is in line with the observation from the graphical analysis. To transform the nonstationary exchange rate series, we obtain daily returns calculated as; $r_{,t} = ln\left(\frac{p_t}{p_{t-1}}\right) \times 100$ where p_t is the daily closing price at time t, for t = 1,2.. and ln is the natural log

Table 1: Unit root test

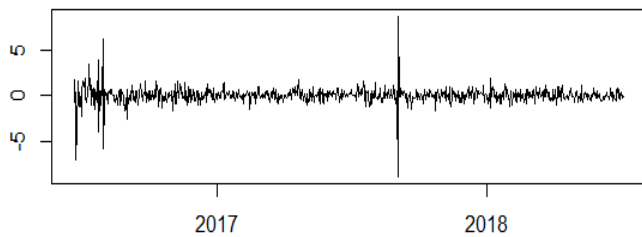| Test	Exchane_data
Augmented Dickey-Fuller	-0.4517
P-Value	0.514
Phillips-Perron Test	-3.0977
P-Value	0.1137

Fig. 2: Return plot of exchange rate.

The Summary Statistics for the returns are shown in table 2. It is noted that the returns series has a mean close to zero, the standard deviation is high which indicates high level of fluctuation in exchange rate return. There is also evidence that the exchange return distribution is leptokurtic and negatively skewed suggesting the exchange rate return is not normally distributed as confirmed by the Jarque-Bera and Shapiro-Wilk tests with P-values less than the critical value, The ADF and PP tests were also applied to the exchange return series both tests indicate that the return series is stationary.

Table 2: Summary Statistics

Statistic	Exchange_ret
Mean	-0.007466741
Std. Dev	0.8896067
Minimum	-8.926138
Maximum	37.59217
Skewness	-0.7774165
Kurtosis	36.38092
Jarque-BeraTest	2436504.9619
P-value	2.2e-16
Shapiro-Wilk Test	0.2658
P-value	2.2e-16
Augmented Dickey-Fuller Test	-12.274
PValue	0.01

A. ARIMA Model fitting–

To start with, the conditional mean process of the exchange rate data is modelled with ARIMA, among different ARIMA models applied, ARIMA (2, 1, 1) appears to be optimal on the basis of possessing minimum AIC and BIC. Estimates of the parameters of the model are shown in table 3, both the coefficients of AR (2) and MA (1) are statistically significant, in addition the forecast performance of the fitted ARIMA model is investigated through mean error (ME), mean absolute error (MAE), root mean square error (RMSE), based on the observed values the forecast performance indicators, the forecasted values are close to the actual values which implies that the model fits the data.

Table 3: Result of ARIMA (2, 1,1)

Parameter	Estimate	Std. Error	z value	Pr(>|z|)
ar1	0.753936	0.045184	16.6859	< 2.2e-16
ar2	0.166168	0.038604	4.3044	1.675e-05
ma1	-0.964640	0.026077	-36.9917	< 2.2e-16
log likelihood	-1996.31			
AIC	4000.62			
BIC	4019.09			
ME	0.1048233			
MAE	1.769403			
RMSE	4.731945			

A. ARIMA Model cheking –

Having chosen the best ARIMA model, the adequacy of the model is checked by analyzing the residuals, we examine whether the squares of residuals are a sequence of white noise and the residuals are homoscedastic of arch effects. The ARCH family of models requires the presence of 'ARCH effect' in the residuals in order to be applied. There are various tests including Portmanteau Q test, Lagrange Multiplier (LM), autocorrelation and partial autocorrelations, Jarque-Bera test, and Ljung-Box test; the Portmanteau Q test and Lagrange Multiplier (LM) test were applied to test the whether the residuals are white noise and the presence of ARCH effect respectively. The diagnostic test result presented in table 5 shows that the model has passed the diagnostic test of normality and evidence of remaining ARCH effect. The confirmation of the presence of ARCH effects indicates that volatility in the exchange return series is time varying, therefore the conditional heteroscedasticity in estimated ARIMA errors is modeled using GARCH model

Table 4: Result of ARMA (2,1) model checking

Test	Test statistic	P
Portmanteau-Q test	**133**	**<0.001**
Lagrange-Multiplier test	**550.3**	**<0.001**

B. GARCH Model fitting –

Among different models applied, GARCH (1, 1) appears to be optimal on the basis of possessing minimum AIC and BIC. The estimates of the parameters of the model are presented in table 5 The estimated parameters are statistically significant.

Table 5: Result of GARCH (1, 1)

Parameter	Estimate	Std. Err	t val	Pr(>|t|)
Mu	-0.16550	0.10167	-1.628	0.10356
Omega	0.52270	0.16262	3.214	0.00131
alpha1	0.24740	0.05233	4.728	2.27e-06
beta1	0.77242	0.03190	24.213	< 2e-16
LogLikelihood	-2.516144			
AIC	5.042983			
BIC	5.067675			
ME	0.07080631			
MAE	0.3264584			
RMSE	0.3891713			

C. GARCH Model checking –

The diagnostic tests results presented in table 6 indicate that the fitted GARCH (1, 1) model has adequately captured the volatility and there is no ARCH effect left in the residuals, also the model has passed the diagnostic test of normality. The forecast performance of the fitted GARCH model is investigated through mean error (ME), mean absolute error (MAE), root mean square error (RMSE), based on the observed values the forecast performance indicators we can say that the forecasted values are close to the actual values and that the model fits the data, hence good for forecasting purpose along with capturing the volatility.

Table 6: Result of GARCH (1, 1) model checking

Test	Statistic	p-Value
Jarque-Bera Test	**44827.78**	**<0.001**
Shapiro-Wilk Test	**0.8284302**	**<0.001**
Ljung-Box Test	**11.8533**	**0.2949949**
LM Arch Test	**1.008097**	**0.9999852**

IV. CONCLUSION

This study focuses on developing the best models to efficiently forecast the exchange rate of Nigerian Naira and Sterling Pounds using ARIMA and GARCH models. Daily exchange rate ranging from June 2016 to July 2019 is obtained from Central Bank of Nigeria for this purpose After the data was subjected to various tests it was discovered that exchange rate series is nonstationary, the return of the exchange series was obtained and appears to be stationary. Further empirical analysis led to choosing ARIMA (2, 1, 1) and GARCH (1, 1) models as the optimal in terms of fitting and forecasting the daily exchange rate of Nigerian Naira and Sterling Pounds.

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