



THE STATIC AND DYNAMIC MODULI OF HARD AND SOFT ROCKS

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Abstract—Static and dynamic moduli of hard and soft rocks, collected from north and central Indian regions, were studied through laboratory investigations. The static elastic modulus were obtained using a standard uniaxial compression test under applied loading, whereas the dynamic elastic modulus values are calculated through ultrasonic measurement of elastic waves' velocities (compressive, v_p and shear, v_s). The chosen rocks for the study are: basaltic variants from central India and gneiss, sandstone from northern India. The calculated values of dynamic modulus were compared with values of static modulus determined in laboratory on samples tested by applied loading for hard and soft rocks. An estimation of static modulus from dynamic modulus for soft rocks is not feasible in comparison with hard rocks, due to difference of magnitude of strains under static applied loadings.

Keywords— ultrasonic measurement, dynamic elastic moduli, hard and soft rock, waves' velocities

I. INTRODUCTION

Elastic constants (Young's modulus & Poisson's ratio) are relevant parameters in understanding deformation behavior of rocks. The static elastic modulus were obtained using a standard uniaxial compression test under applied loading, whereas the dynamic elastic modulus values are calculated through ultrasonic measurement of elastic waves' velocities (compressive, v_p and shear, v_s). The knowledge of differences between static and dynamic moduli, is useful in preliminary stages during the engineering study.

The major problem which arises in trying to relate static and dynamic moduli is that most geological materials do not behave in a perfectly linear elastic, homogeneous, isotropic manner when they are subjected to static loading. As a result of this in most cases, there is a difference between the static and dynamic moduli which are thought to be related to the difference in strain levels at which the two sets of moduli are measured. Thus, while the strain levels of 10^{-6} or less involved in the measurement of the dynamic elastic moduli still allow the material to behave in an elastic manner. The strain levels of 10^{-3} or greater involved in the static testing of the material usually result in permanent deformation of its internal structure and a non-linear stress/strain relationship will be

observed [1]. Moreover, from the long term loading point of view- in stability of slopes/ foundations/mining works etc., the elastic properties are derived from the static modulus. In the opposite side, if the loading is short-termed, for example the rock disintegration at blasting works, earthquakes etc., the determination of dynamic modulus is useful [2-3].

Basically, the presence of soft minerals such as clay minerals and poor cementation cause the rocks to become soft. In sedimentary rocks (sandstone, shale, etc.) and weathering products of crystalline rocks (granite, gneiss, etc.) fall in soft rock category. Several classifications summarized the upper limit for soft rocks at an UCS of 25 MPa [4] where as lower limit set at 0.5MPa to differentiate from soils.

In the present study, the dynamic elastic moduli are compared with static moduli by static applied loads for hard and soft rocks. The selected hard rocks are massive Basalts(MB-I, MB-II, MB-III, MB-IV&MB-V) and banded Gneisses(BG-I & BG-II), whereas soft rocks (of <50 MPa) are sandstones (SS(RB), SS(CG), SS(MG)), Basalt with vesicles (VB-I, VB-II, VB-III) and kyanite schist (Kyn.S).The hard and soft rocks were selected based on strength and weathering nature of rock (macroscopically undisturbed). All the above rock samples are collected from north and central Indian regions.

II. ELASTIC MODULI

A. Static Elastic Moduli

The uniaxial compression tests were performed (2.5L/D) on selected rock samples(NX size). The extent of deformation was recorded by straingauges placed at central portion of sample.

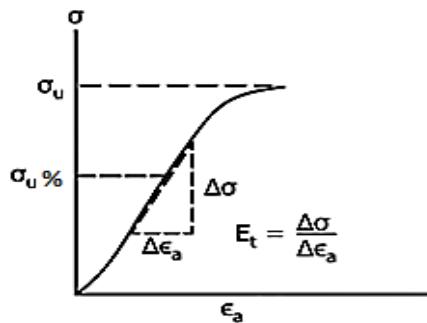


Fig.1 Tangent Modulus (E_t)



From the assumption of CHILE (Continuous Homogeneous Isotropic Linear Elastic) the static modulus E_t was determined by drawing a tangent at 50% ultimate compressive stress(σ) in stress-strain curve (Fig.1), i.e.

$$E_t = \Delta\sigma / \Delta\varepsilon \quad (1)$$

where, σ = axial stress and ε = change in length/the initial length. The calculated values of static modulus E_t and Poisson's ratio, μ for selected specimens of rocks under uniaxial compression is as shown in Table 1.

B. Dynamic Elastic Moduli

Oscilloscope equipment (M/s OYO, Japan) is used for measurement of transmission time of ultrasonic waves' (P-wave and S-wave) for each sample[4]. By virtue of the velocity values of P-wave(V_p) and S-waves(V_s), which reflect the properties of the elastic and isotropic media, it is possible to calculate the dynamic elastic constants such as Young's modulus, shear modulus and Poisson's ratio.

Employing the following basic equations [5-6], in dynamic state, Young's modulus (E_{dyn}) and Poisson's Ratio (μ_{dyn}), are calculated using V_p , V_s and density (ρ):

$$E_{dyn} = \rho V_s^2 (3V_p^2 - 4V_s^2) / (V_p^2 - V_s^2) \quad (2)$$

$$\mu_{dyn} = (V_p^2 - 2V_s^2) / 2(V_p^2 - V_s^2) \quad (3)$$

Since it is a non-destructive method, the waves' velocities are evaluated first for the specimens which are later tested for uniaxial compression by applied loads. Table 1 shows the calculated values of dynamic deformability characteristics.

Table-1 Static and Dynamic deformability characteristics

Rock	γ_{dry} (kg/m ³)	γ_{sat} (kg/m ³)	UCS (MPa)	E_t (GPa)	μ	E_{dyn} (GPa)	μ_{dyn}
Hard Rocks							
MB-I	2810	2870	70	80	0.29	40	0.44
MB-II	2760	2850	70	50	0.25	35	0.35
MB-III	2700	2780	100	65	0.28	29	0.43
MB-IV	2780	2820	80	65	0.29	35	0.41
MB-V	2800	2820	95	65	0.29	75	0.38
BG-I	2700	2710	70	35	0.27	23	0.17
BG-II	2695	2705	65	30	0.23	21	0.02
Soft Rocks							
VB-I	2610	2710	15	30	0.20	24	0.38
VB-II	2580	2670	30	45	0.29	26	0.37
VB-III	2560	2700	18	15	0.29	61	0.26
SS(RB)	2600	2620	50	30	0.15	52	0.37
SS(CG)	2540	2580	48	17	0.26	31	0.12
SS(MG)	2490	2600	13	9	0.27	14	0.33
Kyn S	2730	2740	35	23	0.25	20	0.10

The above table shows the representative values of selected rocks. On an average 8 specimens per each rock were tested to obtain representative value.

III. RESULTS AND DISCUSSION

The values of both moduli are influenced by different conditions under which the static and dynamic tests are performed. In static experiments, the applied loads are of several magnitudes of MPa whereas at dynamic test does not exceed even 100Pa. The loading at uniaxial compression can cause the closing of microcracks that leads to the growth of deformation and consequently to the decreasing of elastic constant. In contrast, dynamic tests are non-destructive where the time of measuring is in several microseconds.

Table 1 show the variation between the representative static and dynamic moduli for hard and soft rocks. The scatter in data values of moduli in soft rocks is substantial in comparison with hard rocks.

In hard rocks (compact and brittle) under static applied loadings, the magnitude of strains are much lower compared to soft rocks (high plastic and ductile). As such the dynamic modulus measured at low strain levels are quite comparable (or having less scatter) with the strain levels of hard rocks under static loadings. Whereas in soft rocks, magnitude of strains is in higher order, hence, the moduli are incomparable.

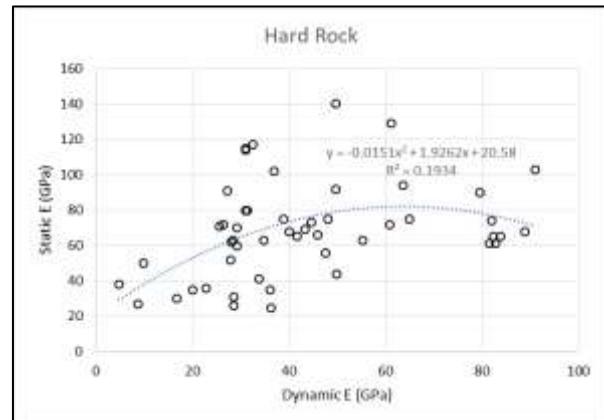


Fig. 2(a) Static and dynamic moduli

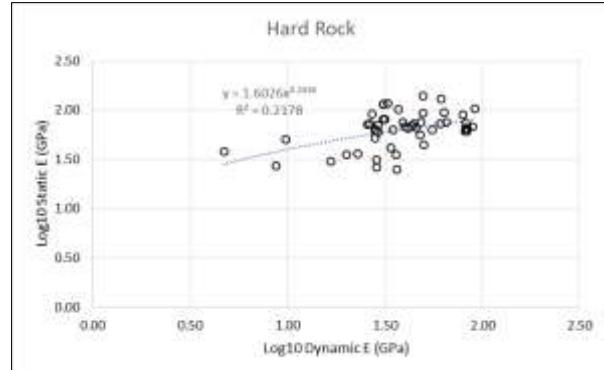


Fig.2(b) Log values Static and dynamic moduli



An attempt has been made to find a relation by comparing the static and dynamic values of Young's modulus for all the tested specimens of hard rocks, is shown in Fig. 2(a) & Fig. 2(b) and the relationship is expressed as:

$$Et = -0.151Edyn^2 + 1.9262Edyn + 20.58 \quad (4)$$

Relationship from logarithmic plot of the same data, Fig. 2(b) is expressed as:

$$\log_{10} Et = 1.6026 \log_{10} Edyn^{0.2545} \quad (5)$$

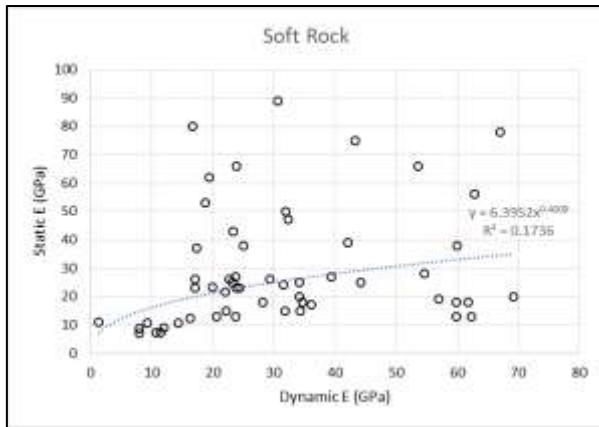


Fig. 3(a) Static and dynamic moduli

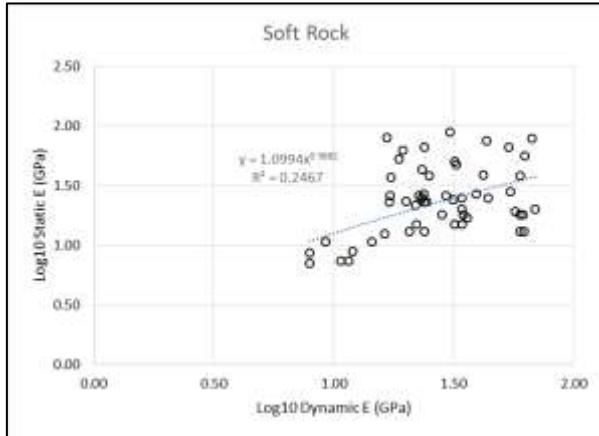


Fig.3(b) Log values Static and dynamic moduli

A similar laboratory study for soft rocks is shown in Fig. 3(a) and the relationship is expressed as:

$$Et = 6.3952 Edyn^{0.4009} \quad (6)$$

Relationship from logarithmic plot of the same data, Fig. 3(b) is expressed as:

$$\log_{10} Et = 1.0994 \log_{10} Edyn^{0.5882} \quad (7)$$

The result shows that some correlation could be determined, it is necessary to continue in this research further by the inclusion of larger data base on hard and soft rocks.

IV. CONCLUSION

The aim of the paper is to show the importance of non destructive dynamic moduli for assessing static modulus of

hard and soft rocks in engineering projects. Based on dynamic moduli obtained through laboratory studies, it was shown that in hard rocks there could be found an analytical solution helping to determine the corresponding static modulus in comparison with soft rocks, in which strains are of higher magnitude under applied static loadings. The measurements and calculation of elastic static modulus have been done on the assumption of the homogeneous isotropic media (non-disturbed or quasi non-disturbed rocks), where the Hook's law is valid; but rocks in general do not fulfil these conditions. The investigation of the space occupied by the objects - samples of rocks - can give an assessment of the real structure of rocks. With the large data base on moduli of hard and soft rocks, the correlations can be further improvised for solving stability problems.

ACKNOWLEDGEMENT

The authors are grateful to Director, CSMRS, for granting permission to publish the work. Thanks are also due to the co-workers from Rock Mechanics Laboratory Division for their help during laboratory works.

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